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« Ouvrages Spéciaux »

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Preface

Reinforced concrete (RC) is the most widely used construction material in civil engineering due to its versatility, strength, and durability. Over time, several types of RC structures have been developed, each serving specific functions and levels of importance. From a structural perspective, some RC constructions — such as retaining walls, water tanks, silos, and bridges — require special design considerations and implementation rules that differ from conventional RC elements. Consequently, civil engineers must possess in-depth technical knowledge to properly design, analyze, and construct these specialized structures.

In this context, the present document has been prepared as a course handout for the *Master 2 – Structures* program. It focuses on special reinforced concrete structures, presenting their theoretical background, design principles, and practical aspects of implementation. The material is aligned with the national academic curriculum and aims to strengthen the student's ability to apply modern design standards to real engineering problems.

To address these objectives, the course handout is organized into five main chapters, each dealing with a specific type of RC structure:

- Chapter 01: Reinforced Concrete Retaining Walls
- Chapter 02: Reinforced Concrete Domes
- Chapter 03: Reinforced Concrete Water Tanks and Towers
- Chapter 04: Reinforced Concrete Silos
- Chapter 05: Reinforced Concrete Bridges

This document serves both as a practical and theoretical guide for Master's students and as a reference manual for practicing engineers. It aims to bridge the gap between academic study and professional application by presenting clear explanations, relevant design procedures, and solved examples that illustrate the design of special reinforced concrete structures according to current engineering standards.

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CHAPTER 01: RC RETAINING WALLS

1. GENERALITIES

1.1.Definition

Retaining structures are constructions intended to prevent the crumbling or sliding (slipping) of a steep embankment. They are mainly employed,

- Either in mountainous areas to protect road surfaces against the risk of landslides or avalanches;
- Either, in an urban site to reduce the footprint of a natural embankment, with a view to the construction of a road, a building or a structure.

There are two main classes of retaining structures.

- Walls which are made up of a resistant wall and a foundation base.
- Screens which are composed only of a resistant wall.

1.2. Behavior of R W

A retaining wall is designed to support a mass of soil and ensure the stability of any structure overlying this soil. We distinguish the excavated wall, which supports excavated soil (excavated embankment), from an embankment wall, supporting artificial soil constructed in compacted layers.

1.3.Role and importance

- A retaining wall is, by definition, a wall intended to contain land in a small space in order to meet various human needs.
- As its name indicates, a retaining wall is used to support, that is to say, to contain and resist the very strong pressures of the ground, most often composed of soil or sand. Generally, it helps combat crumbling and landslides when the ground is sloping.
- Prevent soil erosion

1.4. Types of R W

1.4.1. Cantilever retaining walls (Mur en T renversé)

This solution is only suitable for low heights, i.e., less than 4 m (**Fig. 1.1**).

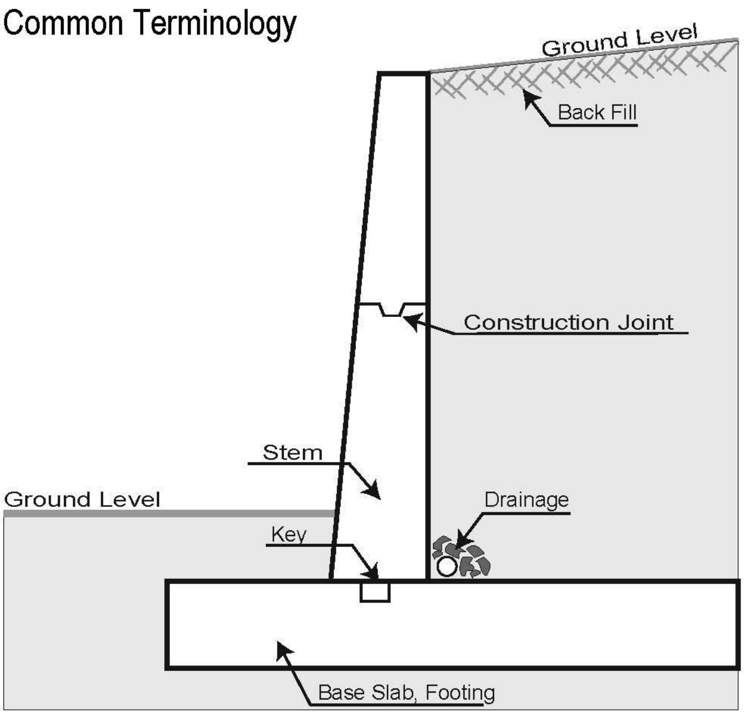


Fig. 1.1. Cantilever retaining wall.

1.4.2. Counterfort Retaining Wall (Mur en T renversé avec contrefort)

This solution is suitable for high heights, i.e., more than 4 m (**Fig. 1.2**).

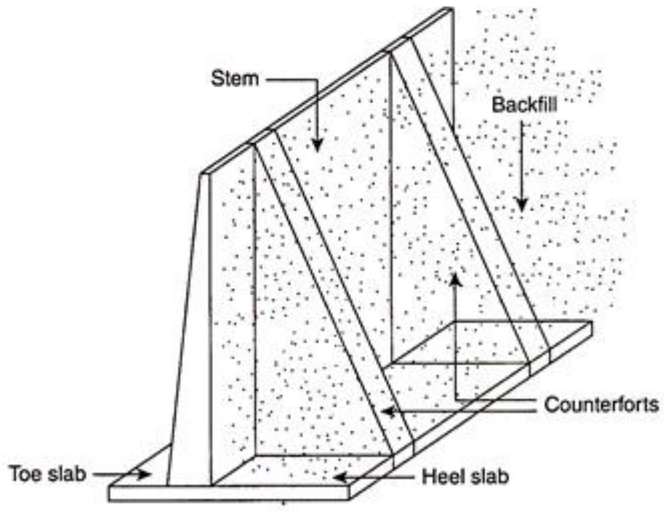
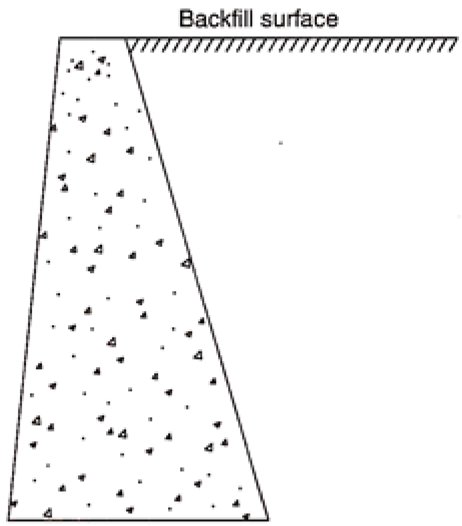


Fig. 1.2. Counterfort retaining wall.

1.4.3. Gravity retaining walls (Mur poids)

Gravity walls derive their lateral stability by their mass (**Fig. 1.3**). Gravity retaining walls are designed to resist earth pressure by their weight. They are constructed of the mass concrete, brick, or stone masonry. Since these materials cannot resist appreciable tension, the design aims at preventing tension in the wall.



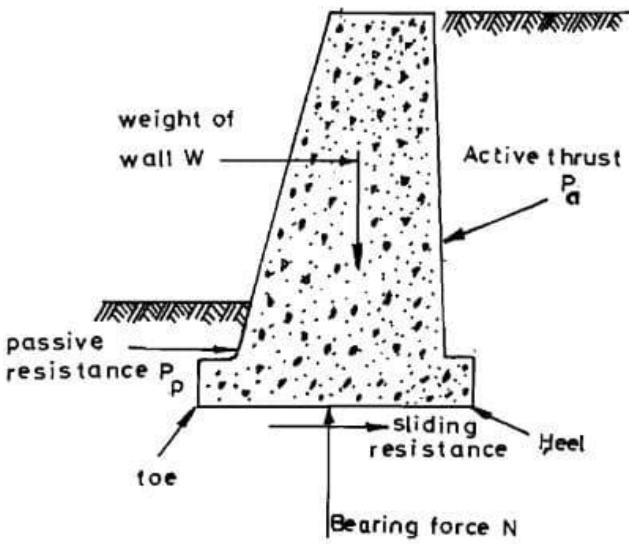


Fig. 1.3. Gravity retaining wall.

1.4.4. Anchored Retaining Wall (Mur ancré)

As it suggests in the name, anchored retaining walls typically resist the active soil pressure forces via an anchor into the soil, rock or other resisting material. These anchors provide the forces necessary to resist overturning and sliding (**Fig 1.4**).

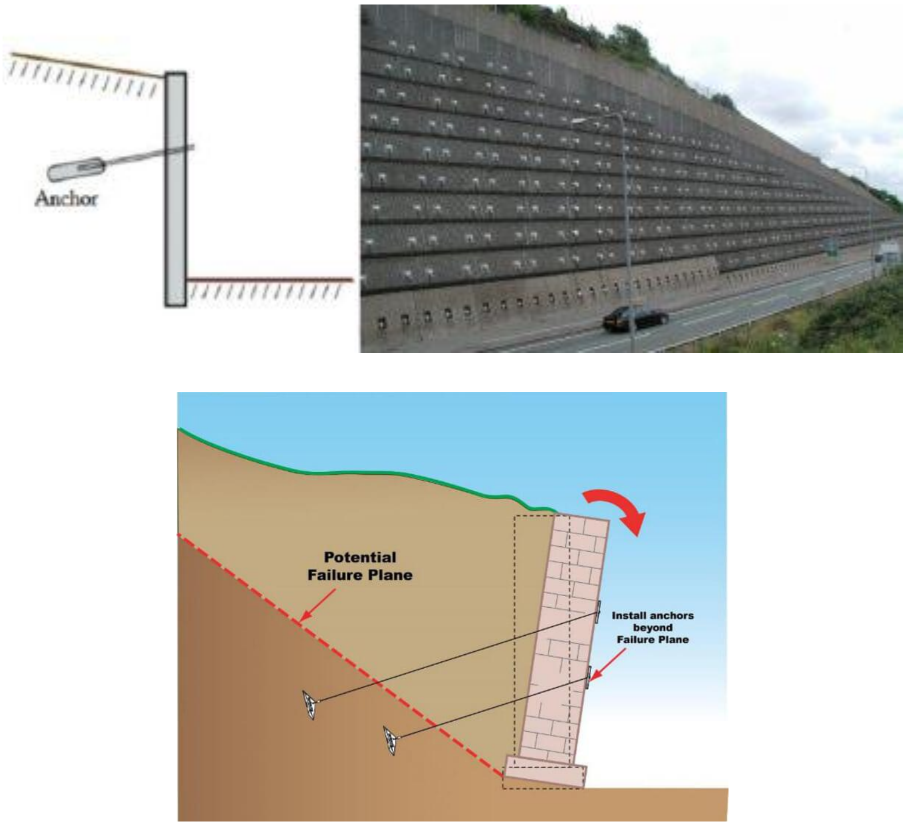


Fig. 1.4. Anchored retaining wall.

1.4.5. Sheet Pile Retaining Walls (palplanches)

Sheet piles (**Fig 1.5**) are flexible retaining structures used to provide a temporary construction area for the construction of structures. Sheet piles are made of timber, steel, or sometimes reinforced concrete. Timber sheet piles were used in the past but their reuse is limited for temporary structures up to shallow depth. For all important structures and for depth > 3 m, steel sheet piles are more commonly used.

Fig. 1.5. Sheet pile retaining wall.

2. STABILITY OF RETAINING WALLS

2.1. External stability

The external stability is verified when the retaining wall resist to sliding, overturning, and soil bearing (**Fig 1.6**).

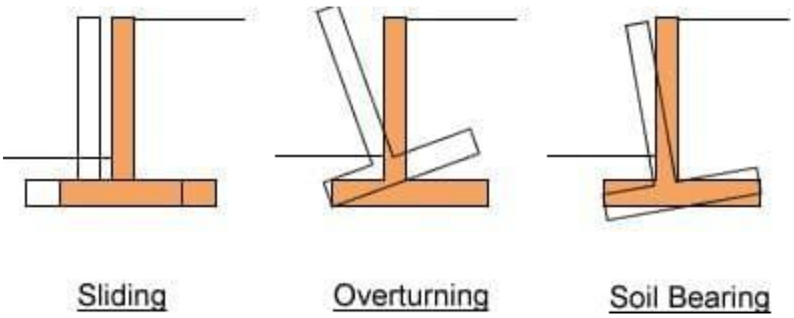


Fig. 1.6. External stability of the retaining walls.

2.1.1. Sliding (Glissement)

$$\text{Factor of safety} = \frac{\sum \text{Friction force}}{\sum \text{Lateral force}} \geq 1.5 \quad (1.1)$$

2.1.2. Overturning (Renversement)

$$\text{Factor of safety} = \frac{\sum \text{Restoring moment}}{\sum \text{Oveturning moment}} \geq 1.5 \quad (1.2)$$

2.1.3. Soil bearing (Poinçonnement)

The stresses diagram under the foundation is shown in **Fig. 1.7**. From this figure, the stress $\sigma_{3/4}$ is given by the following formulas:

$$\sigma_{3/4} = \frac{3\sigma_{max} + \sigma_{min}}{4} \leq \sigma_{soil} \quad (1.3)$$

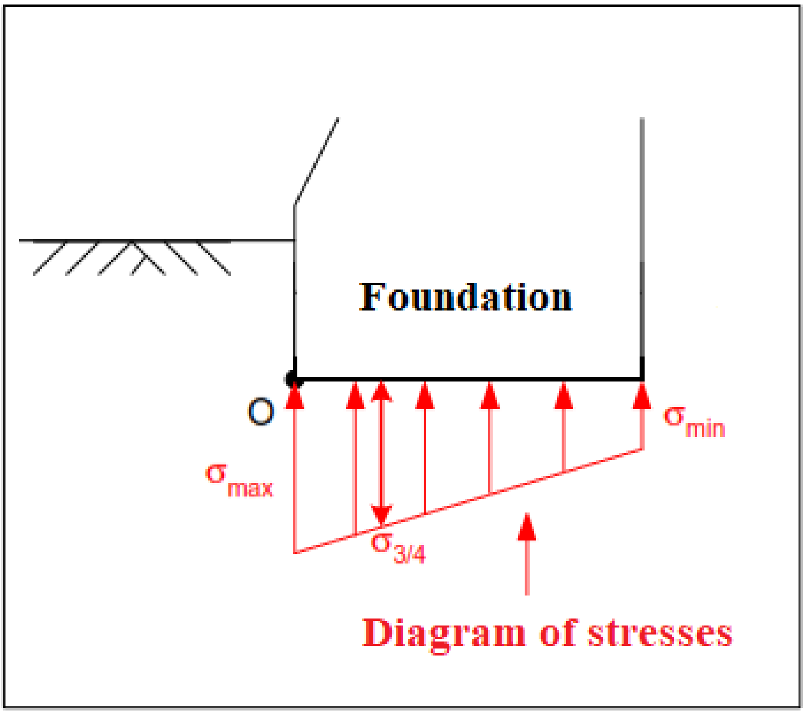


Fig. 1.7. Soil bearing.

2.2. Internal stability

The internal stability is verified by introducing certain quantity of reinforcement. These reinforcements are calculated according to the type of retaining wall. Each element of the wall (Fig 1.8) is calculated according to the acting forces.

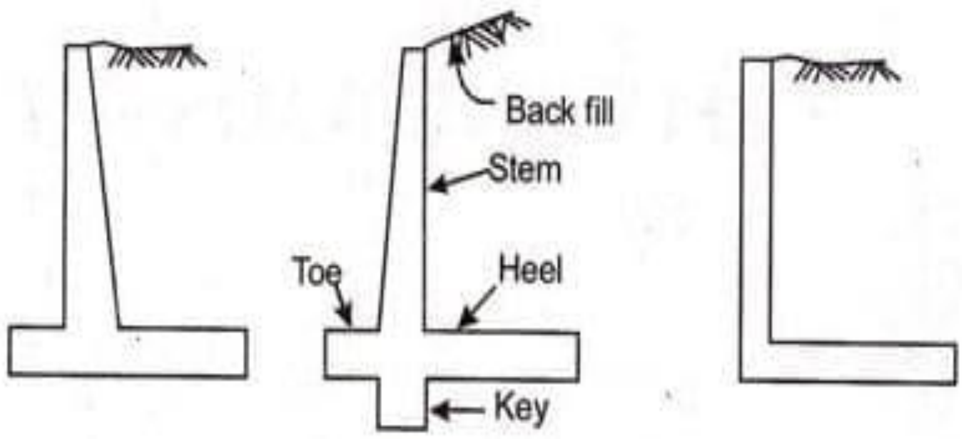


Fig. 1.8. Cantilever RW elements.

2.2.1. Stability of Cantilever RW

2.2.1.1.Pre-dimensioning (preliminary sizing)

Dimensions of the cantilever RW (**Fig 1.9**) are calculated as follows:

- **Stem thickness**

$$\frac{H}{25} < e < \frac{H}{12} \quad with\ e_{min} \geq 20cm \tag{1.4}$$

- **Base slab length**

$$\frac{H}{2} < b < \frac{2 \times H}{3} \tag{1.5}$$

- **Toe length**

$$\frac{H}{8} < b_2 < \frac{H}{5} \tag{1.6}$$

- **Heel length**

$$b_1 = b - (b_2 + e_3) \tag{1.7}$$

- **Base thickness**

$$e_1 > 8\ \%\ (H) \tag{1.8}$$

- **Key thickness**

$$b_3 > 5\ \%\ (H) \tag{1.9}$$

- **Key length**

$$e_3 > \frac{e_1}{2} \tag{1.10}$$

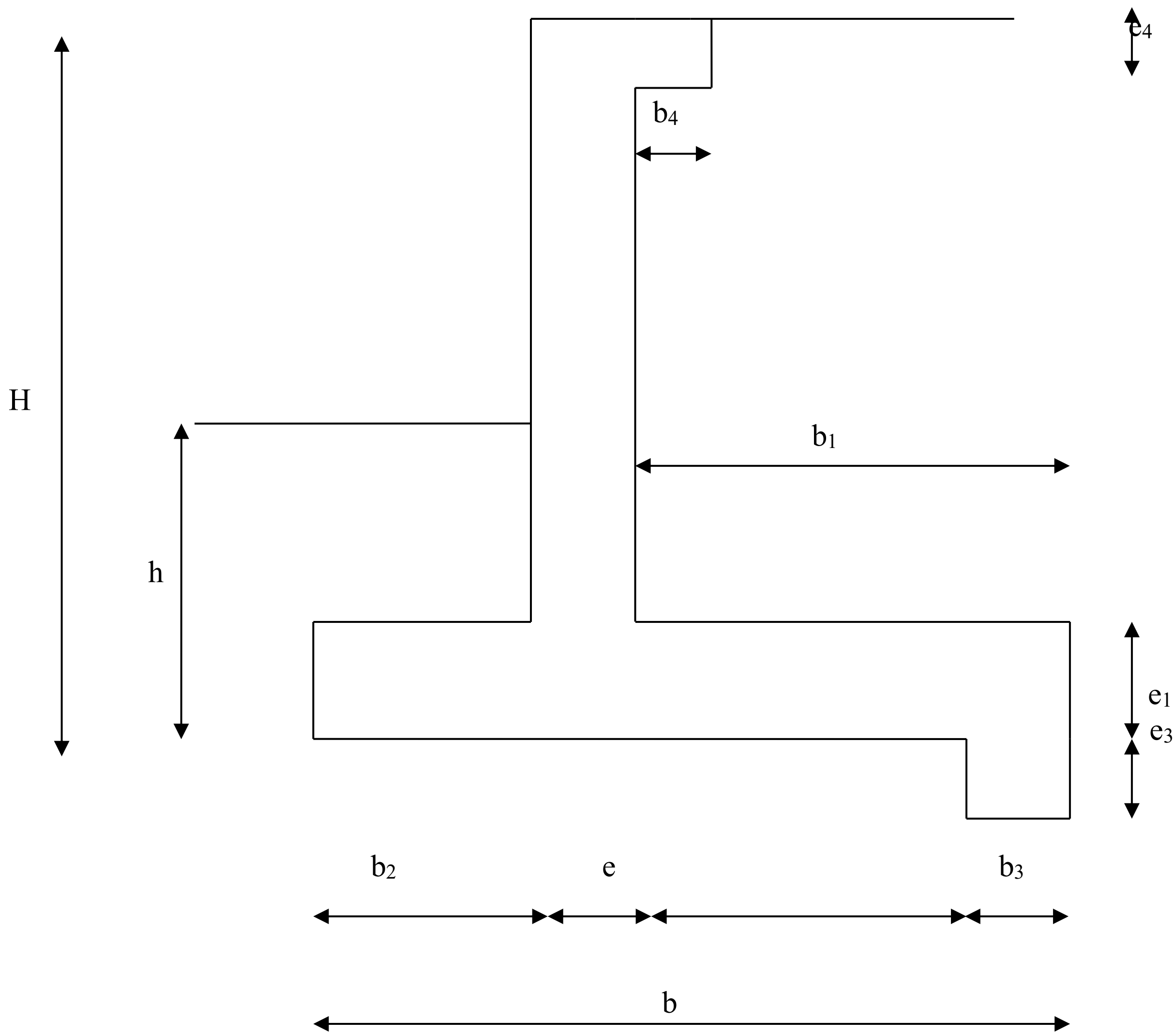


Fig. 1.9. Preliminary sizing.

2.2.1.2.Calculation sections

Analyzing the behavior of a retaining wall requires the calculation analysis of certain sections identified as critical (**Fig. 1.10**).

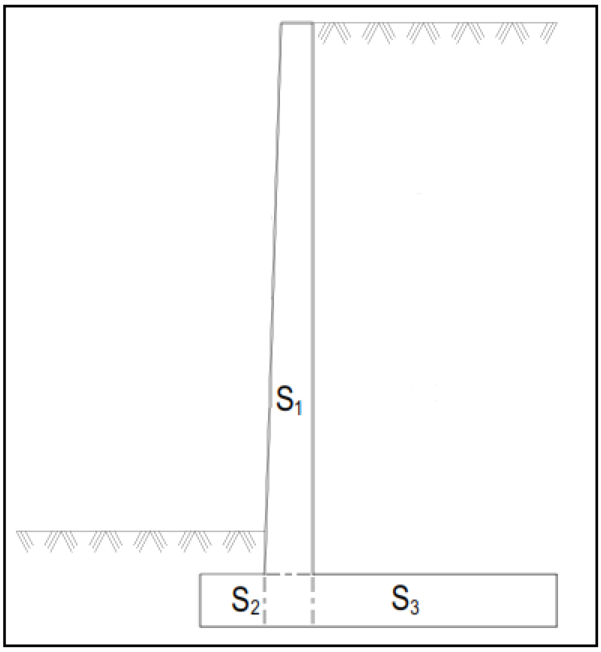


Fig. 1.10. Calculation sections

To determine the necessary reinforcement section in the three cases, sections S1, S2, and S3 are considered as shown in **Table 1.1**:

Table 1.1. Calculation sections.

Section	Definition	Calculation case
S_1	Stem is embedded in the base	Flexure
S_2	Toe is embedded in the stem	Flexure
S_3	Heel is embedded in the stem	Flexure

2.2.1.3.<u>*Disposition of reinforcement*</u>

The main reinforcement of a retaining wall results from the calculation of the critical sections according to the rules of Eurocode 2. **Figs. 1.11 and 1.12** give a typical diagram of the main reinforcement resulting from the calculation.

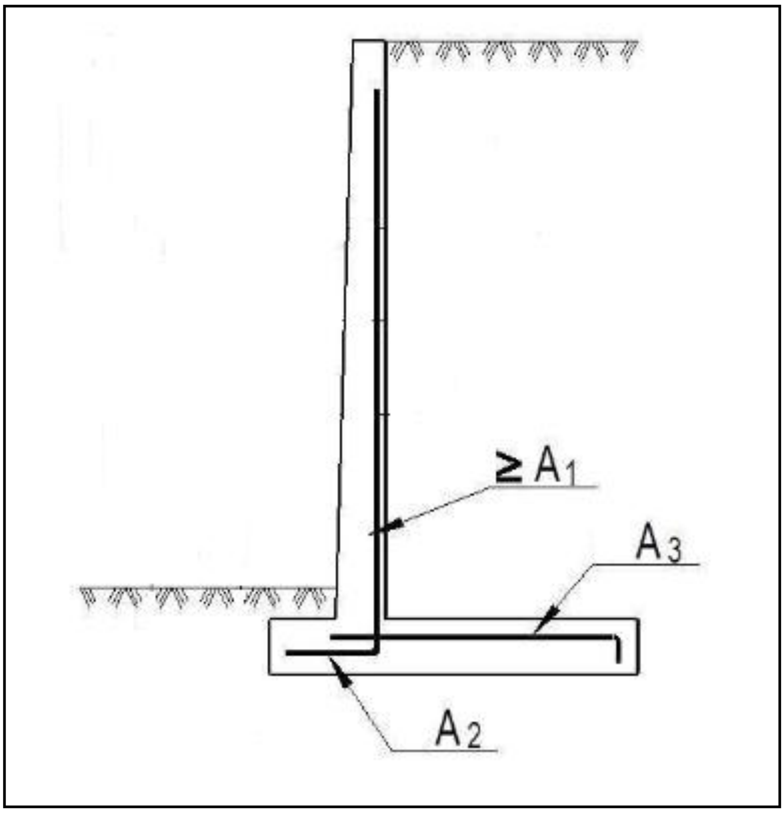


Fig. 1.11. Main reinforcement diagram (1).

2.2.2. Stability of Counterfort RW

2.2.2.1.Pre-dimensioning (preliminary sizing)

Dimensions of the counterfort RW are calculated as follows:

- **Stem thickness**

$$\frac{H}{35} < e < \frac{H}{30} \quad with\ e_{min} \geq 20cm \tag{1.11}$$

- **Counterfort thickness**

$$e_c > 6\ \%\ (H) \tag{1.12}$$

- **Counterfort spacing**

$$\frac{H}{3} < s < \frac{H}{2} \tag{1.13}$$

2.2.2.2. *Calculation*

The stem is calculated in flexure as a two-way slab with beams (slab embedded on three sides) while the heel and the toe are calculated in the same manner as the first case (retaining wall without counterfort).

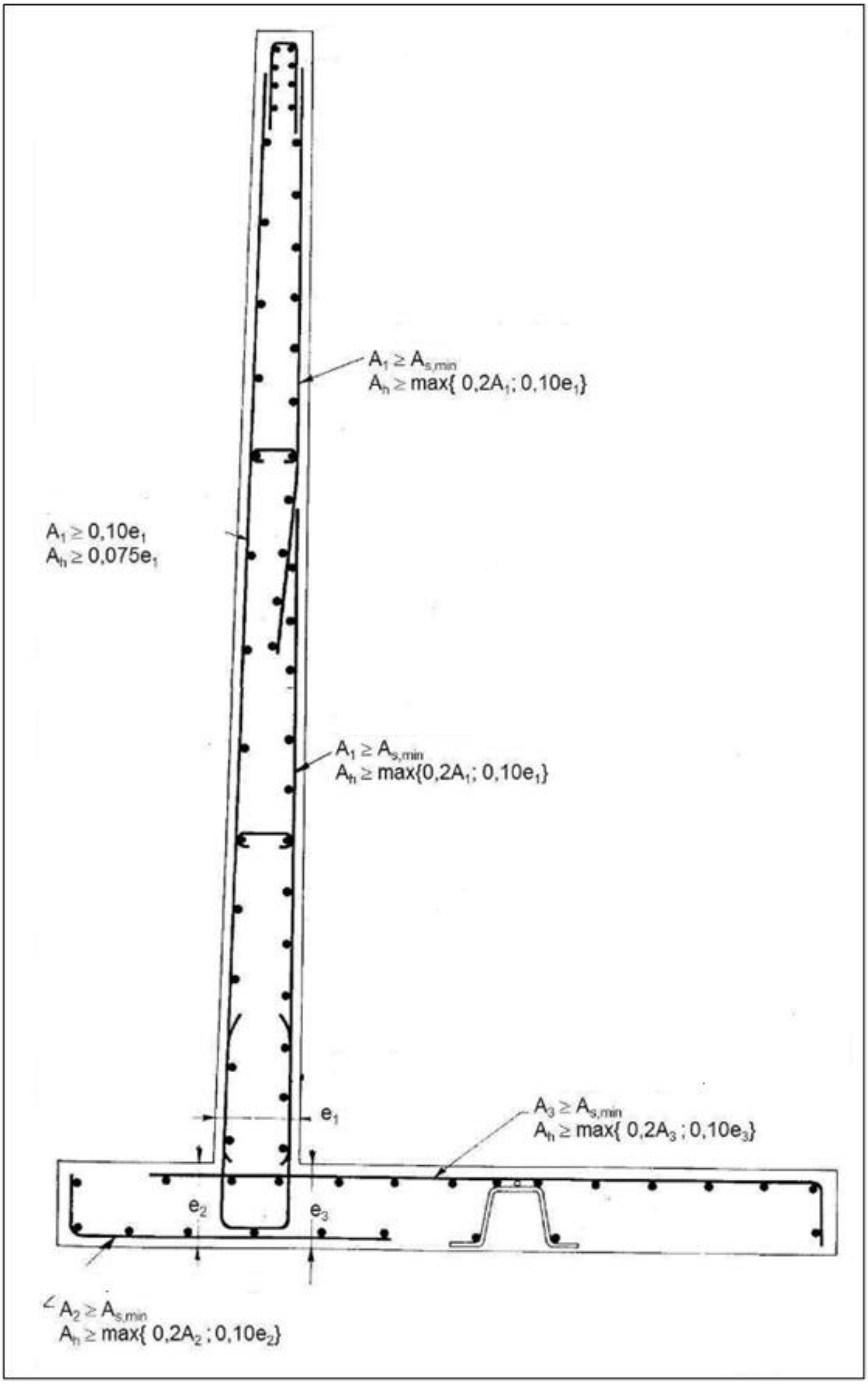
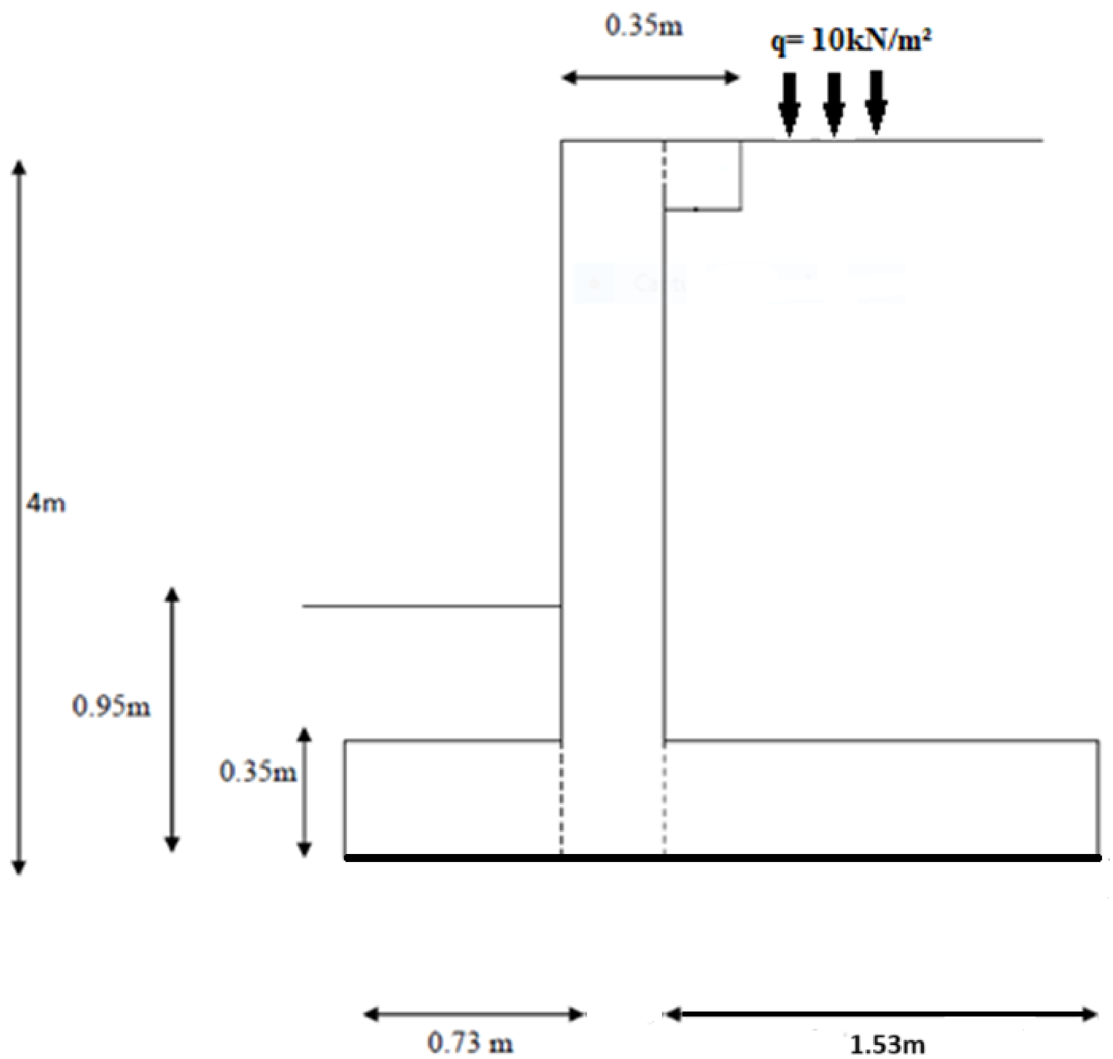


Fig. 1.12. Main reinforcement diagram (2).

3. APPLICATION

- Calculate the necessary reinforcement to ensure the internal stability of the RC cantilever retaining wall shown in **Fig.1.13**. Knowing that:
  - Compressive concrete strength (f'_c) = 25MPa
  - Soil density (γ_s) = 19kN/m^3, coefficient of pressure (K_a) = 0.271.
  - Yield strength of steel reinforcement (f_y) = 400MPa



- Fig. 1.13 -

3.1.Stem

➢ *External moments*

- Moment due to « q »: $M_q = F_q * (h/2) = q \cdot K_a \cdot h \cdot (h/2) = 10 \times 0.271 \times 4 \times 2 = 21.68$ kN
- Moment due to soil: $M_s = F_s * (h/3) = \gamma_s \cdot K_a \cdot h \cdot (h/2) \cdot (h/3) = 19 \times 0.271 \times 4 \times 2 \times (4/3)$ $= 54.92$ kN
- Total moment on stem: $M = 1.35M_s + 1.5M_q = (1.35 \times 54.92) + (1.5 \times 21.68)$ $= 106.66$ kN.m

➢ *Necessary reinforcement*

The stem is calculated in flexure for a strip of one linear meter (b = 1m) as follows:

d = 0.9 x e = 0.9 x 350 = 315mm

$$\mu = \frac{M}{b \times (0.9e)^2 \times \sigma_c} = \frac{106.66 \times 10^3}{1 \times (0.9x350)^2 \times 14{,}17} = 0{,}076 \; ; \mu < \mu_l$$

⇨ $\mu = 0{,}076 < 0{,}392$ ⇒ **compression reinforcement are not necessary $A'_s = 0$.**

$$\Rightarrow \sigma_s = \frac{f_e}{\gamma_s} = \frac{400}{1{,}15} = 348 \text{ MPa}$$

$$\alpha = 1{,}25\left(1 - \sqrt{1-2\mu}\right) = 1{,}25\left(1 - \sqrt{1 - 2x0{,}075}\right) = 0{,}0976$$

$$\beta = 1 - 0{,}4\alpha = 1 - 0{,}4x0{,}0976 = 0{,}961$$

$$z = 0.9 \text{ x e x } \beta = 0.9 \text{ x } 0.35 \text{ x } 0{,}961 = 0{,}303\text{m}$$

$$A_s = \frac{M}{\sigma_s \cdot z} = \frac{106.66x\, 10^3}{348 \text{ x } 0{,}303} = 1011.53\text{mm}^2 = 10.11\text{cm}^2/\text{lm}$$

$$A_{s,min} = \text{MAX}\left(0{,}23 \times \frac{b.d.f_{tj}}{f_e} \; ; \; 0{,}0025 \times e.b\right) = 8.75 \text{ cm}^2/\text{lm}$$

- <u>Spacing</u>

  $S \le min\{2e \,;\, 25cm\}$

  $s \le min\{75 \,;\, 25\ cm\}$

  $S \le 25cm$

- **Adopted reinforcement: 7T14** /lm, with $A_1 = 10.78$ cm^2/lm and $S = 15$cm.

➢ *Verification at serviceability limit state (SLS)*

- *Serviceability moment*

$$M_{ser} = M_s + M_q = 54.92 + 21.68 = 76.6\ \text{kN.m}$$

- *Neutral axis position*

$$\frac{b}{2}y^2 + n(A_s + A'_s)y - n(d.A_s + d'A'_s) = 0$$

Where: A'$_s$ = 0 and n = 15

$$\frac{b}{2}y^2 - nA_s(d-y) = 0 \Rightarrow 50y^2 + 161.7y - 5093.55 = 0 \Rightarrow y = 8.60\ cm.$$

- *Moment of inertia*

$$I = \frac{b}{3}y^3 + n[A_s(d-y)^2 + A'_s(y-d')^2]$$

$$I = \frac{b}{3}y^3 + \eta A_s(d-y)^2 = \frac{100 \times 8.6^3}{3} + [15 \times 10.78(31.5 - 8.60)^2] = 105999\ \text{cm}^4$$

- *Stress in concrete σ_c*

$$\sigma_c = \frac{M_{ser}}{I} \times y = \frac{76.6 \times 10^3}{105999} \times 8.6 = 6.22\ \text{MPa}$$

$$\overline{\sigma_c} = 0{,}6f_{c28} = 15\ \text{MPA.}$$

$\sigma_c = 6.21 < \overline{\sigma_c} = 15$ MPa; **Ok ✓**

- *Stress in steel σ_s*

$$\overline{\sigma_s} = 0.8\ f_y$$

$$\overline{\sigma_s} = 0.8\ x\ 400 = 320\ \text{MPa}$$

$$\sigma_s = \eta\frac{M_{ser}}{I}(d-y) = 1.6\frac{76.6 \times 10^3}{105999}(31.5 - 8.6) = 248.23\ \text{MPa}$$

$\sigma_s = 248.23$ MPa $< \overline{\sigma_s} = 320$ MPa ; **Ok ✓**

➢ *Verification of shear*

V = 1.35V$_s$ + 1.5V$_q$ = [1.35 x γ_s x K$_a$ x h x (h/2)] + [1.5 x q x K$_a$ x h] = [1.35 x 19 x 0.271 x 4 x 2] + [1.5 x 10 x 0.271 x 4] = 55.61 + 16.26 = 71.87 kN

$$\tau_u = \frac{1.5T}{b \times d} = \frac{1.5x71.87 \times 10^3}{1000 \times 315} = 0.34\ \text{MPa}$$

$$\overline{\tau_u} = \min(0{,}15\ f'_c/\gamma_c; 4\text{MPa})$$

$$\overline{\tau_u} = \min(2.5\ ; 4\ ) = 2.5\ \text{MPa.}$$

$\tau_u = 0.34$ MPa $< \overline{\tau_u} = 2.5$ MPa ; **Ok** ✔

➢ *Horizontal reinforcement*

$$A_h = \frac{A_1}{3} = \frac{10.78}{3} = 3.60\ cm^2/lm$$

- **Adopted reinforcement: 5T10** /lm, with $A_h = 3.93\ cm^2/lm$ and $S = 20cm$.

3.2.Heel

The heel is considered as a cantilever. It supports the load "q", the soil weight, and its self-weight. The heel is calculated in flexure for a strip of one linear meter (b = 1m) as follows:

➢ *Moment*

Force due to load q: $F_q = q = 10\ kN/m^2$

Force due to soil: $q_{sh} = \gamma_s$ x (h – 0.35) = 19 x (4 – 0.35) = 69.35 kN/m²

Force due to self-weight: $q_{wh} = e_h$ x $\gamma_{béton}$ = 0.35 x 25 = 8.75 kN/m²

Total force: q_{th} = [1.35 x (F_{wh} + F_{sh})] + (1.5 x F_q) = [1.35 x (8.75 + 69.35)] + (1.5 x 10)

= 120.44 kN/m²

Moment: $M_{th} = q_{th}$ x L_h^2 /2 = 120.44 x (1.53²)/2 = 141 kN.m

➢ *Necessary reinforcement*

d = 0.9 x e = 0.9 x 350 = 315mm

$$\mu = \frac{M}{b \times (0.9e)^2 \times \sigma_{bc}} = \frac{141 \times 10^3}{1 \times (0.9x350)^2 \times 14{,}17} = 0{,}10\ ;\ \mu < \mu_l$$

⇨ $\mu = 0{,}10 < 0{,}392$ ⇒ **compression reinforcement are not necessary A'ₛ =0.**

$$\alpha = 1{,}25\left(1 - \sqrt{1 - 2\mu}\right) = 1{,}25\left(1 - \sqrt{1 - 2x0{,}10}\right) = 0{,}132$$

$$\beta = 1 - 0{,}4\alpha = 1 - 0{,}4x0{,}132 = 0{,}947$$

z = 0.9 x e x β = 0.9 x 0.35 x 0.947 = 0.298m

$$A_s = \frac{M}{\sigma_{st}.\ z} = \frac{141x\ 10^3}{348\ x\ 0{,}298} = 1360mm^2 = 13.60cm^2/lm$$

$$A_{s,min} = MAX\ (0{,}23\ \times\ \frac{b.d.f_{tj}}{f_e}\ ;\ 0{,}0025\ \times\ e.b\ ) = 8.75\ cm^2/lm$$

- Spacing:

$S \leq min\{2e\ ;25cm\}$

$s \leq min\{75\ ;25\ cm\}$

$S \leq 25cm$

- **Adopted reinforcement: 7T16** /lm, with $A_3 = 14.07\ cm^2/lm$ and $S = 15cm$.

➢ *Secondary reinforcement*

$$A_{3h} = \frac{A_3}{3} = \frac{14.07}{3} = 4.70\ cm^2/lm$$

- **Adopted reinforcement: 6T10** /lm, with $A_h = 4.71\ cm^2/lm$ and $S = 15cm$.

3.3. Toe

This part of the foundation is only subject to the reaction of the soil, the weight of the soil above the toe is negligible and could be removed.

	Num.	Efforts		F_h (kN/m)	F_v (kN/m)	Distance (m)	M (kN.m/m)
G		Concrete	Stem		35.00	0.91	31.68
			Heel		13.39	1.85	24.70
			Toe		6.39	0.37	2.33
	(1)	Total			54.78		58.71
	(2)	Soil	Backfill		106.10	1.85	195.76
	(3)		On toe		8.32	0.365	3.04
	(4)	Earth pressure		-41.20		1.33	-54.92
Q	(5)	Weight			15.3	1.85	28.23
	(6)	Pressure		-10.84		2.00	21.70

➢ *Moment*

M_a = 1.35 [(1) + (2) + (3)] + 1.5 [(5) + (6)] + (4) = 302.54 kN.m/lm

F_{vT} = 1.35 [(1) + (2) + (3)] + 1.5 [(5)] = 251.37 kN/lm

Eccentricity: $e_a = M_a / F_{vT}$ = 302.54 / 251.37 = 1.20m

Stress under the toe: $\sigma_t = F_{vT} / 2e_a$ = 251.37 / (2 x 1.20) = 104.43 kN/m²

Moment under the toe: $M_{tt} = \sigma_t \times L_{toe}^2 /2 = 104.43 \times (0.73^2)/2 = 27.83$ kN.m

➢ *Necessary reinforcement*

d = 0.9 x e = 0.9 x 350 = 315mm

$$\mu = \frac{M}{b \times (0.9e)^2 \times \sigma_{bc}} = \frac{27.83 \times 10^3}{1 \times (315)^2 \times 14{,}17} = 0{,}02\ ; \mu < \mu_l$$

⇨ $\mu = 0{,}021 < 0{,}392 \Rightarrow$ **compression reinforcement are not necessary A'_s =0.**

$$\alpha = 1{,}25\left(1 - \sqrt{1 - 2\mu}\right) = 1{,}25\left(1 - \sqrt{1 - 2x0{,}02}\right) = 0{,}025$$

$$\beta = 1 - 0{,}4\alpha = 1 - 0{,}4x0{,}025 = 0{,}99$$

$z = 0.9 \text{ x e x } \beta = 0.9 \text{ x } 0.35 \text{ x } 0.99 = 0.312$m

$$A_s = \frac{M}{\sigma_{st}.\ z} = \frac{27.83x\ 10^3}{348 \text{ x } 0{,}312} = 256.56mm^2 = 2.56cm^2/lm$$

$$A_{s,min} = \text{MAX}\ (0{,}23\ \times \frac{b.d.f_{tj}}{f_e}\ ;\ 0{,}0025\ \times\ e.b\ ) = 8.75\ cm^2/lm$$

- Spacing:

  $S \leq min\{2e\ ; 25cm\}$

  $s \leq min\{75\ ; 25\ cm\}$

  $S \leq 25cm$

- **Adopted reinforcement: 7T14** /lm, with $A_2 = 10.78\ cm^2/lm$ and $S = 15cm$.

➢ *Secondary reinforcement*

$$A_{2h} = \frac{A_3}{3} = \frac{10.78}{3} = 3.60\ cm^2/lm$$

- **Adopted reinforcement: 6T10** /lm, with $A_{2h} = 4.71\ cm^2/lm$ and $S = 15cm$.

3.4.Reinforcement diagram

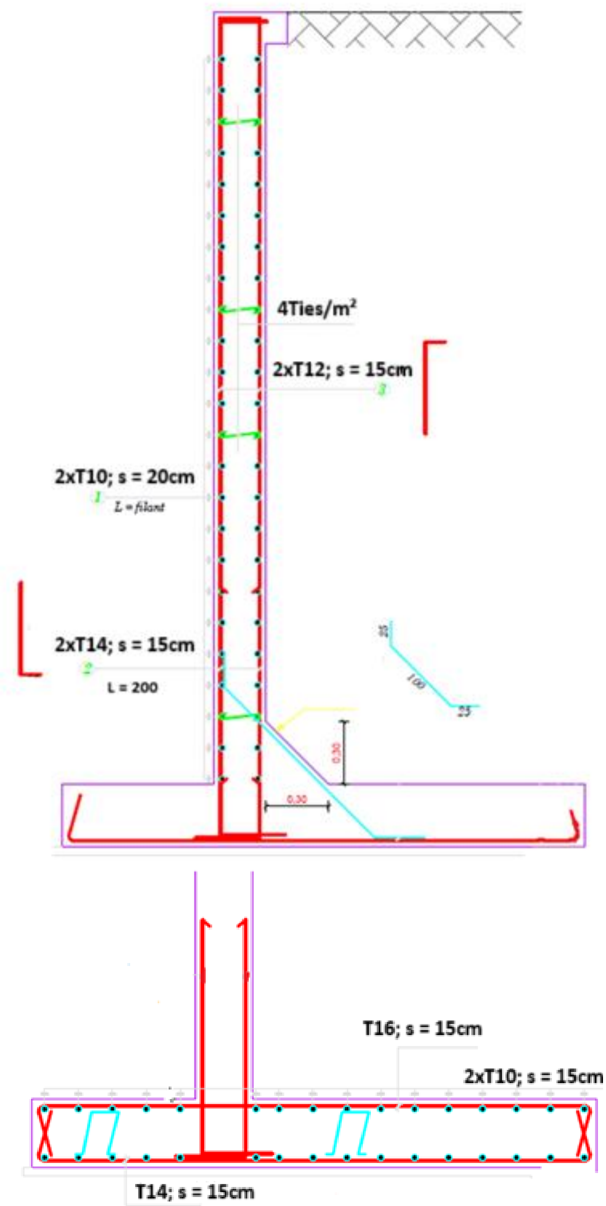


Fig. 1.14. reinforcement diagram

CHAPTER 02: RC DOMES

1. GENERALITIES

1.1.Definition

Domes (**Fig. 2.1**) are shells intended to cover a space, mostly circular, polygonal, or any other shape. Dome is a thin curved surface obtained by revolution of a curved surface about a vertical axis. It is an element of architecture that resembles the hollow upper half of a sphere.

Fig. 2.1. Dome of the Rock (Islamic Architecture).

1.2.Classification of domes

1.2.1. According to the form from plan view

- circular plan.
- elliptic plan.
- polygonal plan.

1.2.2. According to the form of meridian (Fig. 2.2)

- Circular.
- Elliptical.
- Parabolic.
- Ogival.
- Conical.

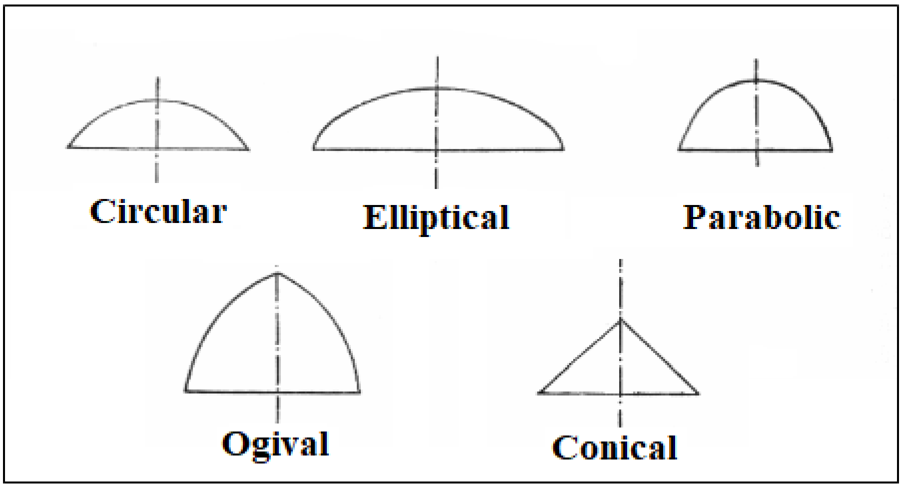


Fig. 2.2. Classification of domes according to the meridian form.

1.2.3. According to the arrangement at the top (Fig. 2.3)

- Closed dome.
- Opened dome.

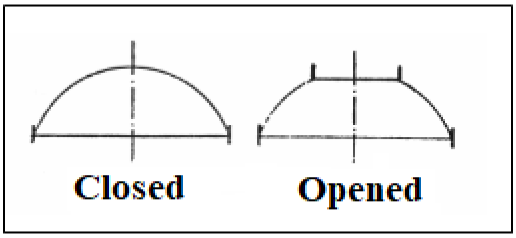


Fig. 2.3. Classification of domes according to the arrangement at the top.

1.2.4. According to the lower hoop (ring) (Fig. 2.4)

- Flexible dome
- Dome with ring beam (hoop compression ring)

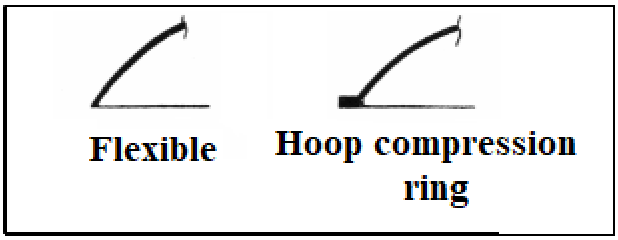


Fig. 2.4. Classification of domes according to the lower hoop.

1.2.5. According to constructive detailing (Fig. 2.5)

- Dome with constant thickness (a).
- Dome with variable thickness (b).
- Dome with meridian ribs (c).
- Dome with parallel ribs (d)
- Dome with orthogonal ribs (e)

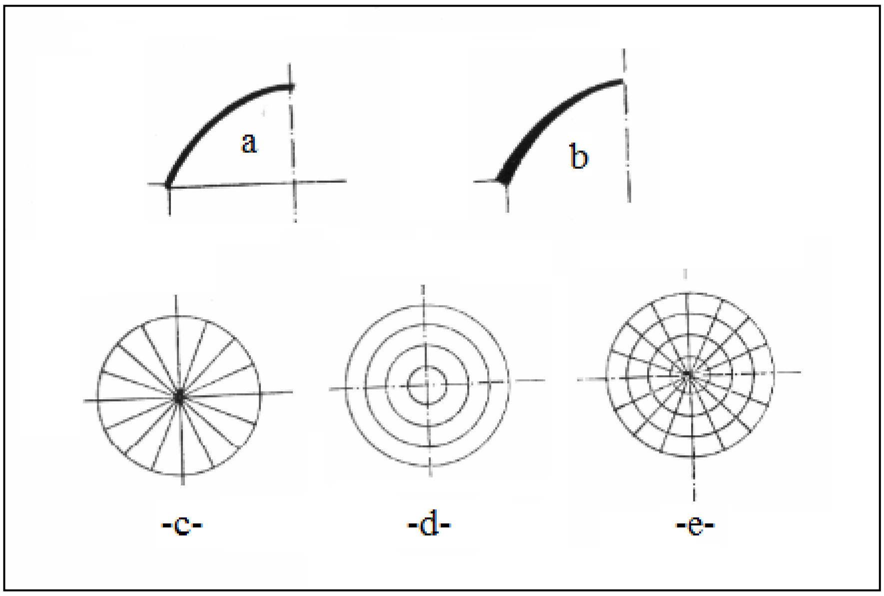


Fig. 2.5. Classification of domes according to constructive detailing.

1.2.6. According to the rise of the dome (Fig. 2.6)

- Surbased dome (diminished).
- Semi-circular dome.
- Surmounted dome (raised)

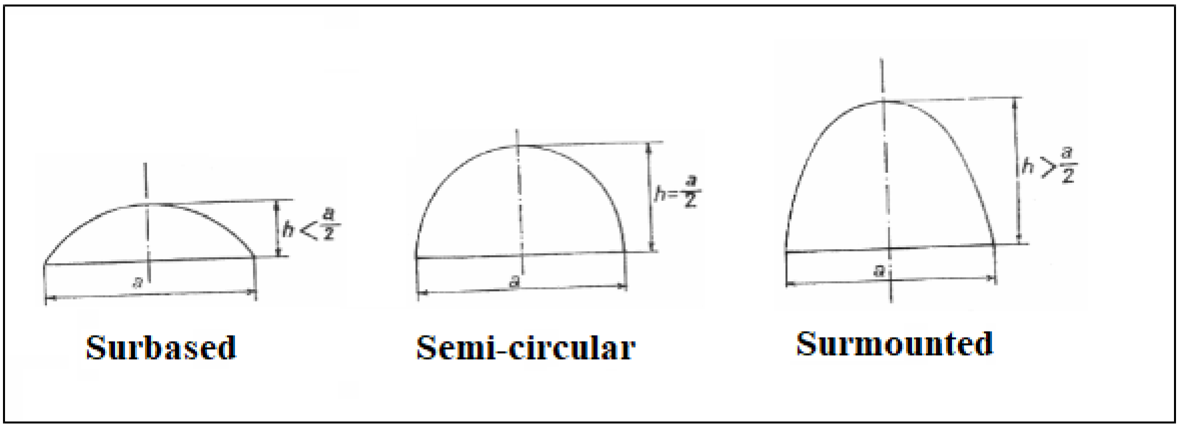


Fig. 2.6. Classification of domes according to the rise of the dome.

2. CALCULATION METHOD
2.1. Membrane method

The membrane method is a rigorous calculation which considering bending (**Fig. 2.7**).

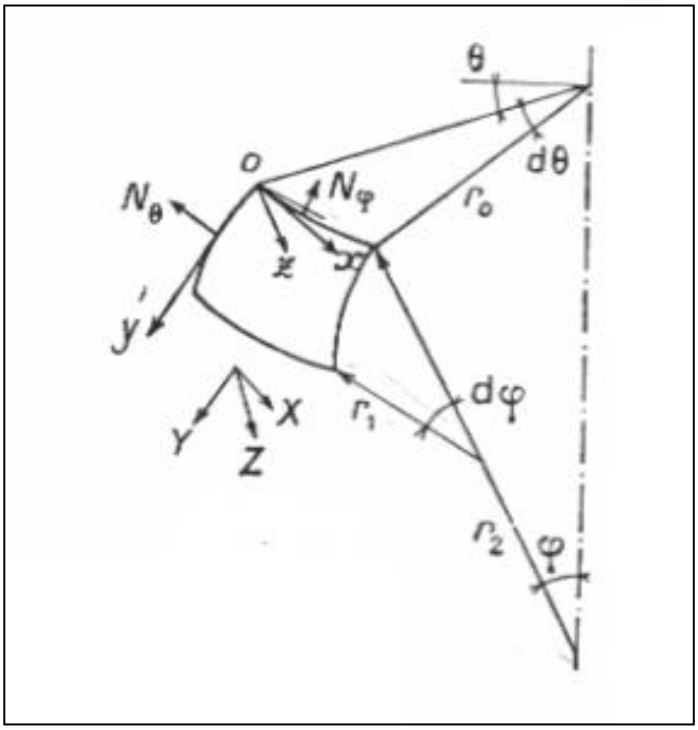


Fig. 2.7. Membrane method.

r_0: curvature radius of the parallel

r_1: curvature radius of the meridian

r_2: curvature radius of the dome

for each type of domes, the internal forces (efforts) are given as follows:

2.1.1. Complete spherical dome with constant thickness (Fig. 2.8)

$$N_\varphi = -\frac{rq}{1+cos\varphi} \quad (2.1)$$

$$N_\theta = rq\left(\frac{1}{1+cos\varphi} - cos\varphi\right) \quad (2.2)$$

Where:

r: radius of the sphere;

q: the applied load per square meter.

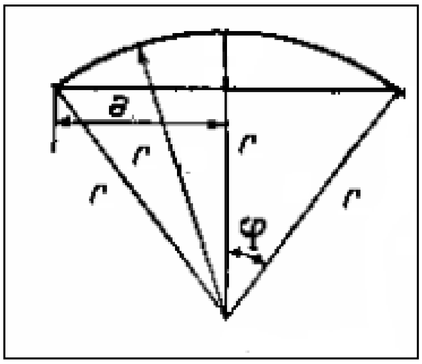


Fig. 2.8. Complete spherical dome.

2.1.2. Complete opened spherical dome with constant thickness (Fig. 2.9)

In the case of a spherical dome with an opening, the internal forces N_φ and N_θ are given as follows:

$$N_\varphi = -rq\frac{\cos\varphi_0 - \cos\varphi}{\sin^2\varphi} - P\frac{\sin\varphi_0}{\sin^2\varphi} \tag{2.3}$$

$$N_\theta = rq\left(\frac{\cos\varphi_0 - \cos\varphi}{\sin^2\varphi} - \cos\varphi\right) + P\frac{\sin\varphi_0}{\sin^2\varphi} \tag{2.4}$$

Where P is the load applied to the opening ring.

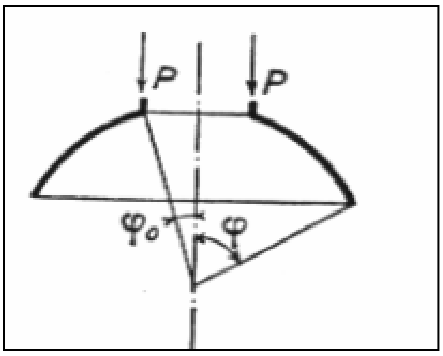


Fig. 2.9. Complete opened spherical dome.

2.1.3. Conical dome with constant thickness (Fig. 2.10)

$$N_\varphi = \frac{P}{2\pi a_0 \cos\alpha} \tag{2.5}$$

$$N_\theta = 0 \tag{2.6}$$

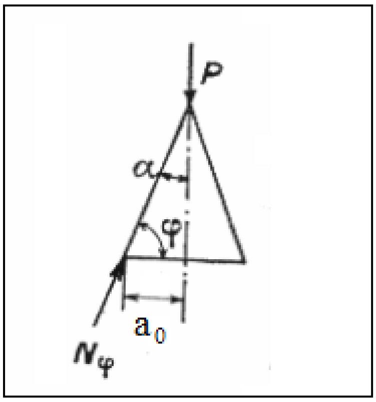


Fig. 2.10. Conical dome.

2.1.4. Ogival dome with constant thickness (Fig. 2.11)

The maximum efforts are occurred as shown in **Table 1**.

$$N_{\varphi} = -rq\frac{(\cos\varphi_0 - cos\varphi) - (\varphi - \varphi_0)\sin\varphi_0}{(sin\,\varphi - \sin\varphi_0)\,sin\,\varphi}\ , N_{\varphi,\max} = -0.793rq \quad (2.7)$$

$$N_{\theta} = -\frac{rq}{sin^2\varphi}[(\varphi - \varphi_0)\sin\varphi_0 - (\cos\varphi_0 - cos\varphi) + (sin\,\varphi - \sin\varphi_0)sin\,\varphi\,cos\varphi]\ ;$$

$$N_{\theta,\max} = +0.522rq \quad (2.8)$$

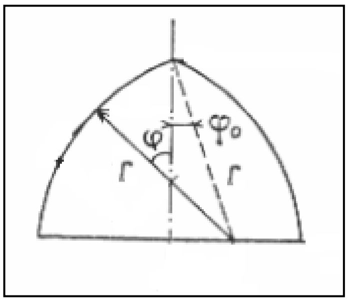


Fig. 2.11. Ogival dome.

2.2. Domes in flexure (bending)

The presence of a rib in a dome introduces a structural anomaly responsible for a modification of the state of membrane stresses since there cannot be equality of deformations, on the one hand in this rib, and, on the other hand, elsewhere, in the immediately neighboring circular ring. This circumstance introduces flexures, especially in a certain area close to the rib. The moments and forces resulting from the theory of shells are given as follows:

$$N_{\varphi} = \frac{E*e}{1-\eta^2}\left[\frac{1}{r_1}\left(\frac{dv}{d\varphi} - \omega\right) + \frac{\eta}{r_2}(v\,cot\varphi - \omega)\right] \quad (2.9)$$

$$N_\theta = \frac{E*e}{1-\eta^2}\left[\frac{1}{r_2}(\upsilon\, cot\varphi - \omega) + \frac{\eta}{r_1}\left(\frac{d\upsilon}{d\varphi} - \omega\right)\right] \quad (2.10)$$

$$M_\varphi = -\frac{E*e^3}{12(1-\eta^2)}\left[\frac{1}{r_1}\frac{d}{d\varphi}\left(\frac{\upsilon}{r_1} + \frac{d\omega}{r_1 d\varphi}\right) + \frac{\eta}{r_2}\left(\frac{\upsilon}{r_1} + \frac{d\omega}{r_1 d\varphi}\right) cot\varphi\right] \quad (2.11)$$

$$M_\theta = -\frac{E*e^3}{12(1-\eta^2)}\left[\frac{cot\varphi}{r_2}\left(\frac{\upsilon}{r_1} + \frac{d\omega}{r_1 d\varphi}\right) + \frac{\eta}{r_1}\frac{d}{d\varphi}\left(\frac{\upsilon}{r_1} + \frac{d\omega}{r_1 d\varphi}\right)\right] \quad (2.12)$$

Where: E: modulus of elasticity of the material; e: thickness of the dome; η: Poisson's ratio; r1: radius of curvature of the meridian; r2: radius of curvature of the dome; υ: tangential displacement of the meridian; ω: normal displacement of the meridian.

Table 1- Where maximum forces occur in ogival domes.

Force type	**Location of maximum**	**Value**
Meridional compression (N_φ)	Near springing (90° ≤ ϕ ≤ 100°)	Shell carries vertical load mainly in meridian direction
Hoop force (N_θ)	Near springing (90° ≤ ϕ ≤ 100°)	Geometry causes large circumferential thrust

2.3.Buckling of domes

The buckling of a dome is an instability phenomenon that is rarely observed, given the rigidity introduced by the double curvature. Generally, failure by compression, tension, or shear precedes failure by buckling. For a uniformly compressed spherical dome the critical buckling stress is given by the following relation:

$$\sigma_{cr} = \frac{E*e}{r\sqrt{3(1-\eta^3)}} \quad (2.13)$$

2.4. Practical method for calculating sur-based spherical domes

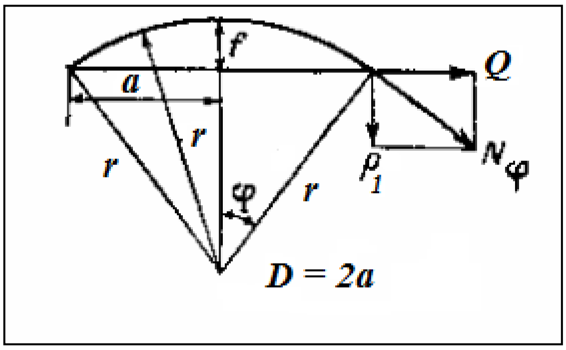


Fig. 2.12. Sur-based dome.

The following simplified method can be used for the calculation of sur-based domes (**Fig. 2.12**). The radius of curvature is given as follows:

$$r = \frac{D^2+4f^2}{8f} \tag{2.14}$$

The total load is given by the following as follows:

$$p = q\frac{\pi D^2}{4} \tag{2.15}$$

Where q is the uniformly distributed load per m2 of the horizontal projection (self-weight + overload).

Dividing the dome into spindles of 1 m long around the perimeter, the weight of each spindle will be:

$$p_1 = q\frac{D}{4} \tag{2.16}$$

The thrust will be given by:

$$Q = q\frac{D^2}{24f} \tag{2.17}$$

The normal internal force N_φ will be equal to:

$$N_\varphi = \sqrt{{p_1}^2 + Q^2} \tag{2.18}$$

The tangential tensile force of the ring will be equal to:

$$F = Q\frac{D}{2} \tag{2.19}$$

2.5.Membrane method (case of asymmetrical loads)

We previously dealt with the domes under symmetrical loads, but it can also be a question of asymmetrical loads (**Fig. 2.13**). For a spherical shell ($r_1 = r_2 = r$), the internal forces N_θ, N_φ, and $N_{\theta\varphi}$ are given as follows:

$$N_\varphi = -\frac{rq}{3}\frac{\cos\theta\cos\varphi}{\sin^3\varphi}\left(2 - 3\cos\varphi + \cos^3\varphi\right) \tag{2.20}$$

$$N_\theta = \frac{rq}{3}\frac{\cos\theta}{\sin^3\varphi}\left(2\cos\varphi - 3\sin^2\varphi - 2\cos^4\varphi\right) \tag{2.21}$$

$$N_{\theta\varphi} = \frac{rq}{3}\frac{\sin\theta}{\sin^3\varphi}\left(2 - 3\cos\varphi - \cos^3\varphi\right) \tag{2.22}$$

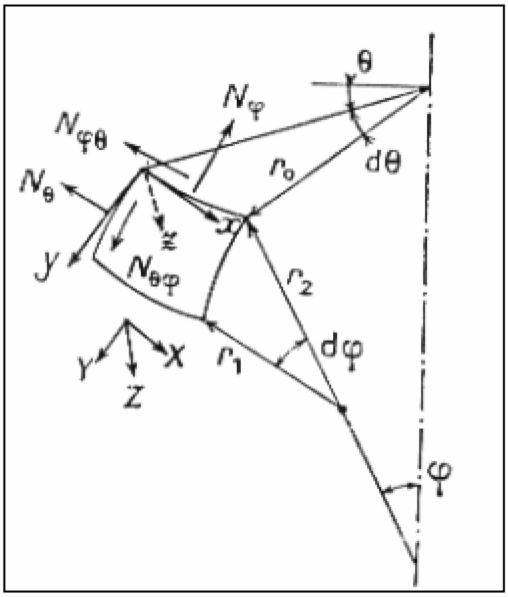


Fig. 2.13. Dome with asymmetrical loading.

3. APPLICATION

Consider an RC dome (A) with a diameter D_A = 15m and a radius of curvature R_A = 10.9m. This dome supports another RC dome (B) with a diameter D_B = 5m, a radius of curvature R_B = 2.5m, and an arrow *f* = 2.5m (see **Fig. 2.14)**.

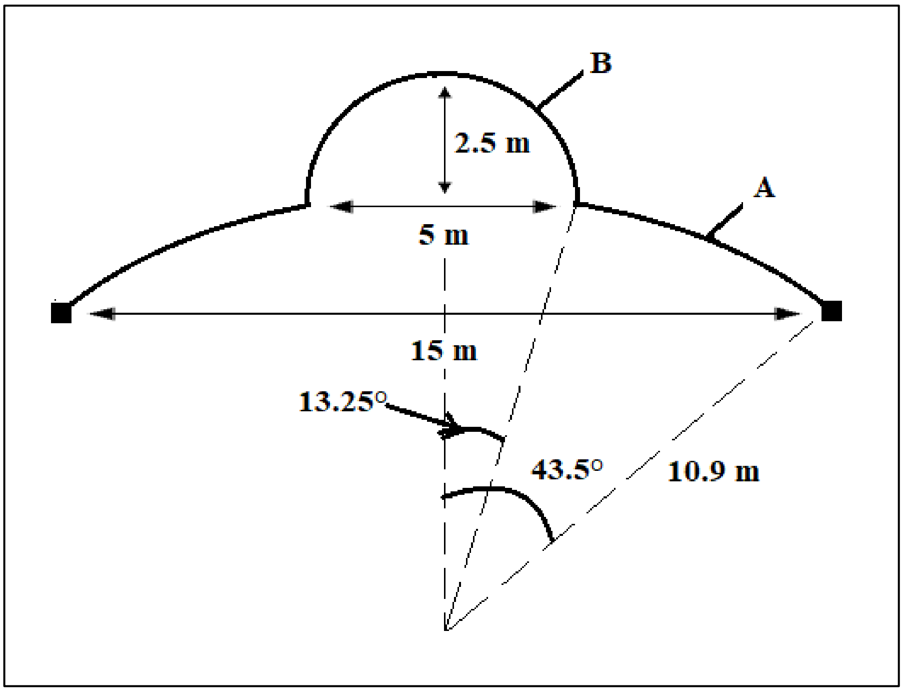


Fig. 2.14

- Calculate the necessary reinforcement to ensure the internal stability of this RC dome. Knowing that:
    - Live load: q = 100 daN/m^2.
    - Concrete thickness (e_A) = 10cm.
    - Compressive concrete strength = 25MPa.
    - Yield strength of steel reinforcement = 400MPa.
    - RC density = 25kN/m^3.

3.1.Dome B "complete spherical dome"

Total load: $Q_u = 1.35G + 1.5q = 1.35x2.5 + 1.5x1 = 4.88kN/m^2$

➢ $N_\varphi = -\frac{rQ_u}{1+cos\varphi} = -\frac{2.5\ x\ 4.88}{1+cos90} = -12.2kN\ (compression)$

Stress in concrete: $\sigma_c = \frac{N_\varphi}{e\ x\ 1lm} = \frac{12.2\ x\ 10^3}{100\ x\ 1000} = 0.122MPa\ < \overline{\sigma_c}(0{,}6f'_c = 15\ MPA)$

$A_{s,min}$ = MAX (4 cm^2; 0,2% × $e.b$) = (4 cm^2; 2 cm^2) = 4.00 cm^2/lm

- Spacing:

$S \leq min\{e + 10\ ; 40\ cm\}$
$s \leq min\{\ 20\ ; 40\ cm\}$
$S \leq 20cm$

- **Adopted reinforcement: 6T10** /lm, with $A_{B\varphi} = 4.71cm^2/lm$ and $S = 15cm$.

➢ $N_\theta = rQ_u\left(\frac{1}{1+cos\varphi} - cos\varphi\right) = 2.5\ x\ 4.88\left(\frac{1}{1+cos90} - cos90\right) = +12.2\ kN\ (Tension)$

$$A_s = \frac{N_\theta}{\sigma_{st}} = \frac{12.2x\ 10^3}{348} = 35.05mm^2 = 0.35cm^2/lm$$

$$A_{s,min} = A_c\frac{f_{ct}}{f_y} = 10\ x\ 100\frac{2.1}{400} = 5.25\ cm^2/lm$$

- Spacing:

$S \leq min\{e\ ; b\ cm\}$
$s \leq min\{10\ ; 100\ cm\}$
$S \leq 10cm$

- **Adopted reinforcement: 10T10** /lm, with $A_{B\theta} = 7.85cm^2/lm$ and $S = 10cm$.

3.2.Upper ring

Volume of sphere: $V = \frac{4}{3}\pi r^3$

Volume of dome B: $V_B = \frac{2}{3}\pi({r_1}^3 - {r_2}^3) = \frac{2}{3}\pi(2.5^3 - 2.4^3) = 3.77m^3$

Perimeter of upper ring: $P_B = 2\pi r = 2\ x\ \pi\ x\ 2.5\ = 15.7m$

Weight of dome B: $W_B = V_B x\ 25 = 94.25kN$

Surface of dome B: $S_B = 2\pi rf = 2\ x\ \pi\ x\ 2.5\ x\ 2.5 = 39.25m^2$

Total Surcharge q on dome B: $q_B = q \text{ x } S_B = 1\ x\ 39.5 = 39.25\ kN$

Total charge of dome B: $Q_{uB} = 1.35W_B+1.5q_B = (1.35x94.25) + (1.5x39.25) = 186.11kN$

Load applied to the opening ring: P = 186.11 / 15.7 = 11.85kN/lm

Horizontal pressure: $H' = \frac{P(r^2-f^2)}{2xfxr} = \frac{11.85(2.5^2-2.5^2)}{2x2.5x2.5} = 0\ kN$

Normal effort: $N = \sqrt{H'^2 - P^2} = \sqrt{0^2 - 11.85^2} = 11.85\ kN$

Tension effort: $T = \frac{P(r^2-f^2)}{2xfxr}\text{r} = \frac{11.85(2.5^2-2.5^2)}{2x2.5x2.5}2.5 = 0\ kN$

Stress in concrete: $\sigma_c = \frac{N}{e\ x\ 1lm} = \frac{11.85 \text{ x } 10^3}{100\ x\ 1000} = 0.12MPa < \overline{\sigma_c}(0{,}6f'_c = 15 \text{ MPA})$

$A_{s,min} = \text{MAX}\ (4 \text{ cm}^2;\ 0{,}2\% \times e.b\ ) = (4 \text{ cm}^2;\ 2 \text{ cm}^2\ ) = 4.00 \text{ cm}^2/\text{lm}$

The use of starter reinforcement (bar) in the connection area between the two domes is sufficient.

3.3.Dome A "complete opened spherical dome"

- **<u>Upper edge</u>**

$\varphi = \varphi_0 = 13.25°$

- $N_\varphi = \frac{-P}{sin\varphi} = \frac{-11.85}{sin13.25} = -51.70kN\ (compression)$

Stress in concrete: $\sigma_c = \frac{N_\varphi}{e\ x\ 1lm} = \frac{51.70 \text{ x } 10^3}{100\ x\ 1000} = 0.52MPa < \overline{\sigma_c}(0{,}6f'_c = 15 \text{ MPA})$

- $N_\theta = -RQ_u cos\varphi + \frac{P}{sin\varphi} = -10.9x4.88xcos13.25 + \frac{11.85}{sin13.25} = -51.76 + 51.7 = -0.06kN\ (compression)$

- ***Lower edge***

- $N_\varphi = -RQ_u \frac{\cos\varphi_0 - cos\varphi}{sin^2\varphi} - P\frac{\sin\varphi_0}{sin^2\varphi}$

$$= -10.9\ x4.88 \frac{\cos 13.25 - cos43.5}{sin^2 43.5} - 11.85 \frac{\sin 13.25}{sin^2 43.5}$$

$$= -27.84 - 5.73 = -33.57kN\ (Compression)$$

Stress in concrete: $\sigma_c = \frac{N_\varphi}{e\ x\ 1lm} = \frac{33.57 \text{ x } 10^3}{100\ x\ 1000} = 0.34MPa < \overline{\sigma_c}(0{,}6f'_c = 15 \text{ MPA})$

$A_{s,min} = \text{MAX}\ (4 \text{ cm}^2;\ 0{,}2\% \times e.b\ ) = (4 \text{ cm}^2;\ 2 \text{ cm}^2\ ) = 4.00 \text{ cm}^2/\text{lm}$

- <u>Spacing:</u>

$S \leq min\{e + 10\ ; 40\ cm\}$

$s \leq min\{20\,;40\ cm\}$

$S \leq 20cm$

- **Adopted reinforcement: 6T10** /lm, with $A_{A\varphi} = 4.71cm^2/lm$ and $S = 15cm$.

➤ $N_\theta = RQ_u\left(\frac{\cos\varphi_0 - cos\varphi}{sin^2\varphi} - cos\varphi\right) + P\frac{\sin\varphi_0}{sin^2\varphi}$

$= 10.9\ x4.88\left(\frac{\cos 13.25 - \ cos43.5}{sin^2 43.5} - cos43.5\right) + 11.85\frac{\sin 13.25}{sin^2 43.5}$

$= -10.74 + 5.73 = -5.01kN\ (Compression)$

Stress in concrete: $\sigma_c = -\frac{N_\theta}{e\ x\ 1lm} = \frac{5.01\ x\ 10^3}{100\ x\ 1000} = 0.05MPa\ < \overline{\sigma_c}(0{,}6f'_c = 15\ MPA)$

$A_{s,min} = MAX\ (4\ cm^2;\ 0{,}2\%\ \times\ e.b\ ) = (4\ cm^2;\ 2\ cm^2\ ) = 4.00\ cm^2/lm$

- Spacing:

$S \leq min\{e + 10\,;40\ cm\}$

$s \leq min\{20\,;40\ cm\}$

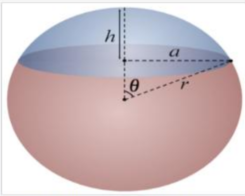
$S \leq 20cm$

- **Adopted reinforcement: 6T10** /lm, with $A_{A\theta} = 4.71cm^2/lm$ and $S = 15cm$.

3.4.Lower ring

Volume of spherical cap:

	Using r and h	Using a and h	Using r and θ
Volume	$V = \frac{\pi h^2}{3}(3r - h)$ [1]	$V = \frac{1}{6}\pi h(3a^2 + h^2)$	$V = \frac{\pi}{3}r^3(2+\cos\theta)(1-\cos\theta)^2$
Area	$A = 2\pi rh$[1]	$A = \pi(a^2 + h^2)$	$A = 2\pi r^2(1-\cos\theta)$
Constraints	$0 \leq h \leq 2r$	$0 \leq a,\ 0 \leq h$	$0 \leq \theta \leq \pi,\ 0 \leq r$



An example of a spherical cap in blue (and another in red)

The height of dome A: $f_A = r - \sqrt{r^2 - a^2} = 10.9 - \sqrt{10.9^2 - 7.5^2} = 3m$

Surface of dome A: $S_A = (2\pi rf) - (\pi {r_1}^2) = (2\pi\ x\ 10.9\ x\ 3) - (\pi 2.5^2) = 205.36 - 19.63 = 185.73m^2$

Weight of dome A: $W_A = V_A x\ 25 = S_A x\ e_A\ x\ 25 = 185.73\ x\ 0.1\ x\ 25 = 464.33kN$

Total weight of dome A+B: $W_T = W_A + W_B = 464.33 + 94.25 = 558.6kN$

Total Surcharge q on dome A+B: $q_T = q\ x\ (S_A + S_B) = 1\ x\ (185.73 + 39.25)\ = 225\ kN$

Total charge of dome A+B: $Q_{uT} = 1.35W_T + 1.5q_T = (1.35x558.6) + (1.5x225) = 1092kN$

Perimeter of lower ring: $P_A = 2\pi r = 2\ x\ \pi\ x\ 7.5\ = 47.1m$

Load applied to the opening ring: P = 1092 / 47.1 = 23.20kN/lm

Horizontal pressure: $H' = \frac{P(r^2 - f^2)}{2xfxr} = \frac{23.20(10.9^2 - 3^2)}{2x3x10.9} = 39\ kN$

Normal effort: $N = \sqrt{H'^2 - P^2} = \sqrt{39^2 - 23.20^2} = 31.35\ kN$

Stress in concrete: $\sigma_c = \frac{N}{A_c} = \frac{31.35 \times 10^3}{400 \times 400} = 0.20MPa \ < \overline{\sigma_c}(0{,}6f'_c = 15 \text{ MPA})$

Tension effort: $T = \frac{P(r^2-f^2)}{2xfxr} r = \frac{23.20(10.9^2-3^2)}{2x3x10.9} 10.9 = 425.1 \ kN$

Or:

$H = N_{max} cos\varphi = 33.57 cos43.5 = 24.35 \ kN$

$T = H \frac{D}{2} = 24.35 \frac{15}{2} = 182.63 \ kN$

$\overline{\sigma_{st}} = \min\left(\frac{2f_e}{3}; 90\sqrt{\eta f_{tj}}\right) = \min(266.66; 90\sqrt{1.6x2.1}) = 165MPa$

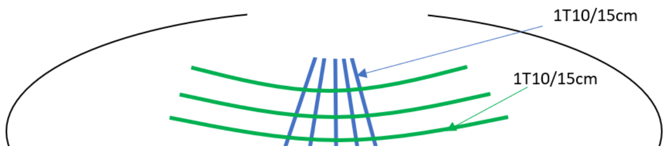
$A_s = \frac{T}{\overline{\sigma_{st}}} = \frac{425.1x\ 10^3}{165} = 2576.36mm^2 = 25.76cm^2/lm$

$A_{s,min} = A_c \frac{f_{ct}}{f_y} = (40 \text{ x } 40)\frac{2.1}{400} = 8.4 \text{ cm}^2/lm$

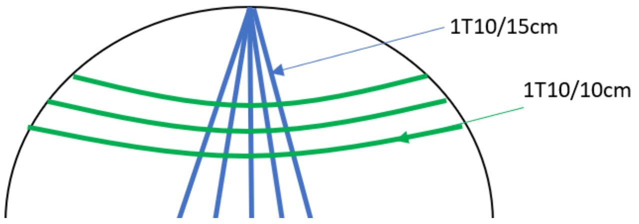
- **Adopted reinforcement: 4T20 + 8T16** /lm, with $A_{slr} = 28.65cm^2/lm$.

3.5.Reinforcement diagram

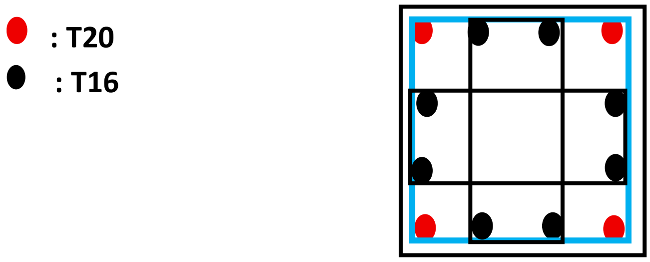
- Dome A



- Dome B



- Lower ring



CHAPTER 03: RC WATER TANKS AND TOWERS

1. GENERALITIES

1.1.Definition

A tank is an envelope that contains a liquid. In general, this liquid is water. These tanks can either be placed on the ground, slightly buried, on a superstructure (swimming pool), on high pylons (water tower), or above buildings.

1.2.Form from plan view (plan shapes)

The plan shape can be any. However, most of the time small tanks are square or rectangular, although the circular shape is the least expensive for the following reason:

- For a similar section (S), the perimeter (P) of a tank is equal to:
    - Circular:

$$P = \sqrt{4\pi S} = 3.57\sqrt{S} \quad (3.1)$$

    - Square:

$$P = 4\sqrt{S} \quad (3.2)$$

    - Rectangular (S= a*b; b=k*a):

$$P = \frac{2(k+1)\sqrt{k}}{k}\sqrt{S} = \alpha\sqrt{S} \quad (3.3)$$

where:

k	1	2	3	4
α	4	4.23	4.61	5

1.3.Technical requirements

An RC water tank must meet the following requirements:

- Strength: RC tanks must resist to different forces.
- Water tightness (waterproofing): RC tanks must be uncracked and leak-free.
- Sustainability: the concrete must retain its initial properties after extended contact with the contained liquid.

1.4.Effects to take into account

They are of different natures:

- Self-weight (dead loads) of the tank and its annexes.
- Load due to the contained liquid.
- Imposed loads (live loads): from 1kN/m² up to 4.5kN/m².
- Temperature variation.
- Influence of shrinkage.
- Creep intervention.
- Climatic effects (snow and wind).
- Earthquake influence.

1.5.Tanks classification

Tanks may be classified according to:

1.5.1. Position of the tank relative to the ground (Fig.3.1)

- On columns or pylons (elevated or overhead water tanks)
- On buildings

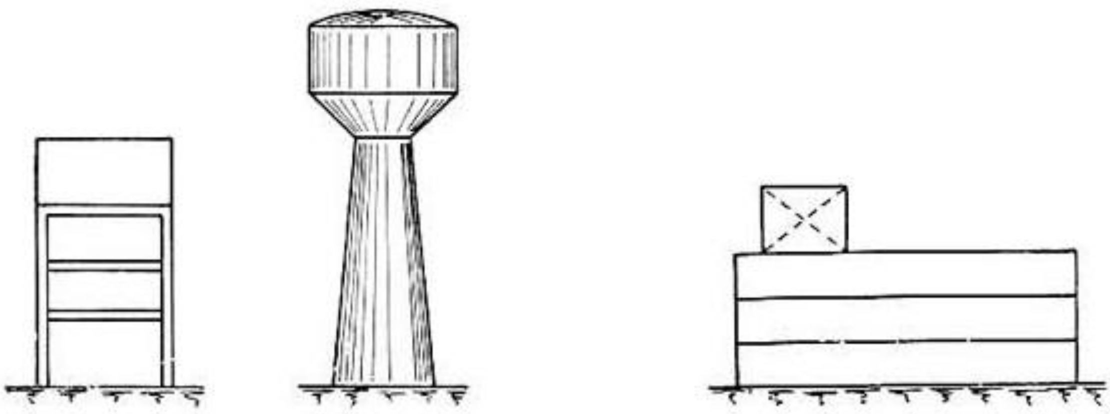


Fig. 3.1

1.5.2. Form of the tank

- Square tank.
- Rectangular tank.

Fig. 3.2

- Circular tank.

Fig. 3.3

- Other shape.

1.5.3. Closing method

- Covered tank
- Uncovered tank

Fig. 3.4

1.5.4. Construction complexity

- Simple tanks.
- Multiple tanks.
- Stacked tanks.
- Stacked and multiple tanks.

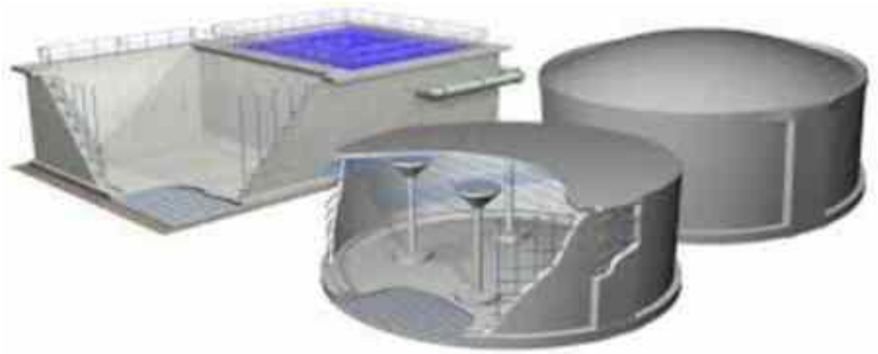


Fig. 3.5

Fig. 3.6

1.5.5. Use

- Storage tanks for different liquids.
- Treatment basins (for water purification).
- Sports basins (swimming pool).
- Gasholder tank.

1.5.6. Nature of stored liquid

- Water tanks and other liquids.
- Black product tankers (bitumen and tar).
- Hydrocarbon tanks.

2. CALCULATION METHODS

2.1.Rectangular water tank placed on the ground

2.1.1. Case of an open tank of small plan dimensions and an important height

In this case, the calculation will be done by assimilating the tank to a rectangular frame subjected to water pressure (**see Fig. 3.7**).

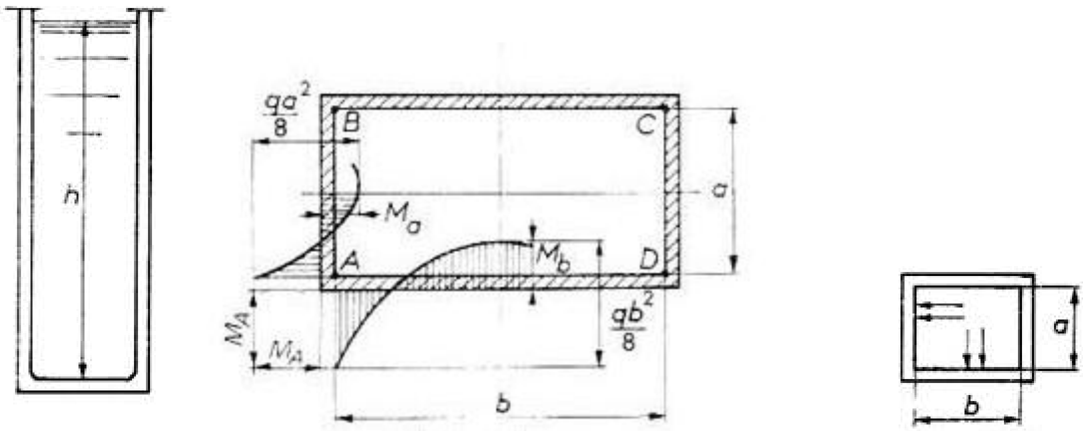


Fig. 3.7

- The moment at the corners:

$$M_A = \frac{-q}{12} + \frac{a^3+b^3}{a+b} \tag{3.1}$$

- The moments on the faces:

$$M_a = \frac{qa^2}{8} - M_A \tag{3.2}$$

$$M_b = \frac{qb^2}{8} - M_A \tag{3.3}$$

The walls can have unequal inertia. Let I_1 be in direction b and I_2 in direction a. With:

$$k = \frac{I_2}{I_1} \times \frac{b}{a} \tag{3.4}$$

2.1.2. Case of a covered tank elongated in plan

- When the tank is placed on good ground, and its cover is loaded uniformly:

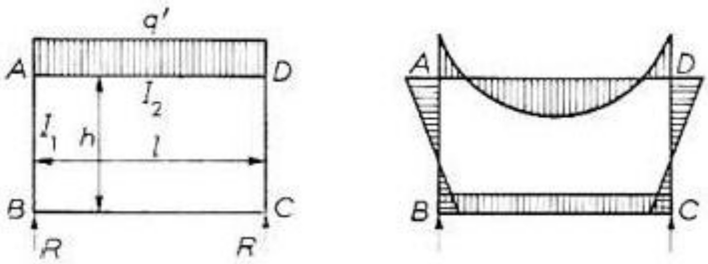


Fig. 3.8

- The moment at the corners

$$M_A = M_D = -\frac{q'l^2}{12}\frac{2k+3}{k^2+4k+3} \tag{3.5}$$

$$M_B = M_C = +\frac{q'l^2}{12}\frac{k}{k^2+4k+3} \tag{3.6}$$

- The moments resulted from the hydrostatic force

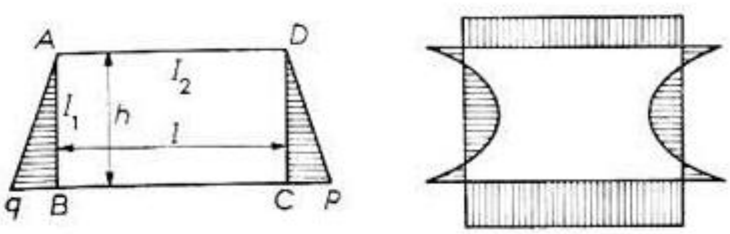


Fig. 3.9

$$M_A = M_D = -\frac{qh^2}{60} \times \frac{k(2k+7)}{k^2+4k+3} \tag{3.7}$$

$$M_B = M_C = -\frac{qh^2}{60} \times \frac{k(3k+8)}{k^2+4k+3} \tag{3.8}$$

- The moment transmitted to the raft foundation:

$$M = \frac{ql^2}{12}\frac{k}{k^2+4k+3} \tag{3.9}$$

- Cover

The cover can be considered as an RC slab or a dome according to their shape.

- Raft foundations

The total full tank weight must be taken into consideration such as:

1. The cover weight and the live load on it.
2. The weight of walls.
3. The weight of the raft foundation.
4. The weight of the contained liquid.

2.1.3. Reinforcement arrangements

1. Wall supported without fixing along its entire perimeter (**Fig. 3.10**)

The reinforcement consists of a grid placed on the outside of the wall, both in the base slab and in the top slab, along the anchorage length. The spacing between the steel bars must not exceed 20 cm. The distribution (secondary) reinforcement must have a cross-sectional area at least equal to one quarter of that of the main reinforcement. Around openings or inspection ports that are sometimes provided in the walls (e.g., wine tanks), the reinforcement is strengthened to form frames or lintels.

It is recommended to use steel bars of relatively small diameter to improve bond strength. The more numerous the bars, the finer and more evenly distributed any cracking will be. Water tightness is therefore improved.

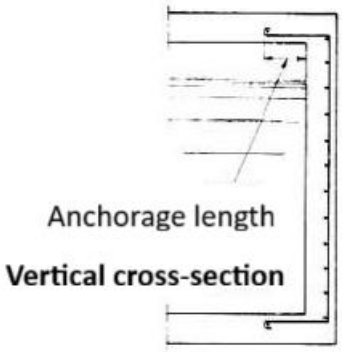


Fig. 3.10

2. Wall fixed on three sides and hinged at the top **(Fig. 3.11)**

The same reinforcement arrangement as in the previous case, with additional bars placed inside the tank along the vertical walls and at the junction with the base slab (**fig. 3.11**) to achieve the required fixity.

Reinforcements located near the outer faces and bent inward can also help to balance the bending moments at the corners.

3. Wall fixed on three sides and free at the top

The reinforcement system is the same as in the previous case, except for its position, as is obvious.

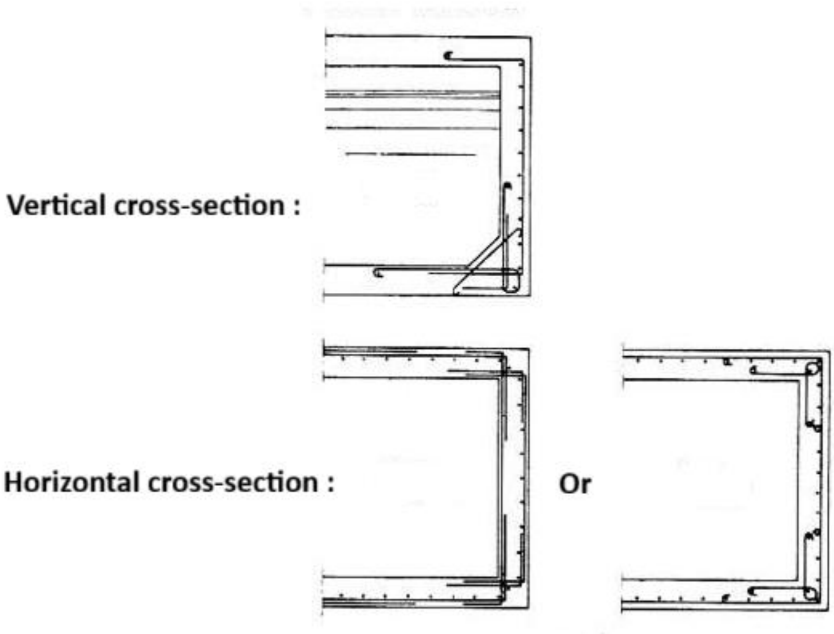


Fig. 3.11

4. Wall fixed on four sides

An outer reinforcement grid is provided together with additional bars to achieve the required fixity.

Theoretically, the bars of the outer grid must have at least the anchorage length and extend beyond the inflection point of the bending moment curve. In practice, every second bar is nevertheless continued up to the corners. It is also necessary to provide the required section to balance the shear force.

5. Other support systems for flat walls

It is easy to take inspiration from the preceding cases.

6. Wall of an uncovered tank of large area and low height

The wall acts as a cantilever fixed into the base slab. The grid formed by the main vertical bars is placed on the inner face (**fig. 3.12**). Additional reinforcement should be provided in the gussets.

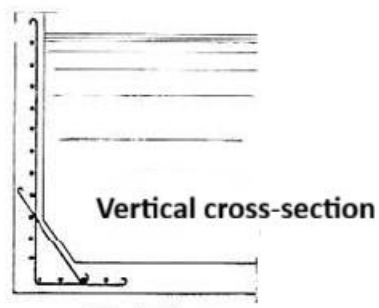
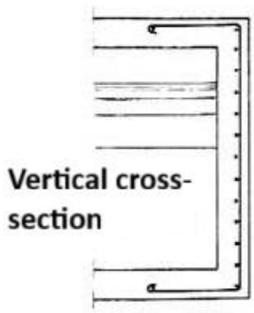


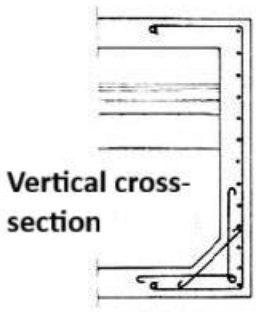
Fig. 3.12

7. Wall of a covered tank of large area and low height

Below are shown the reinforcement layouts for the following cases:

- Two hinges (**fig. 3.14**),
- One hinge and one fixed edge (**fig. 3.15**),
- Two fixed edges (**fig. 3.16**).





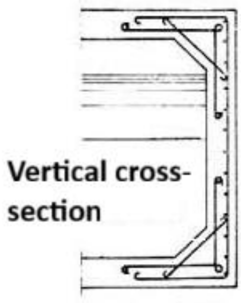
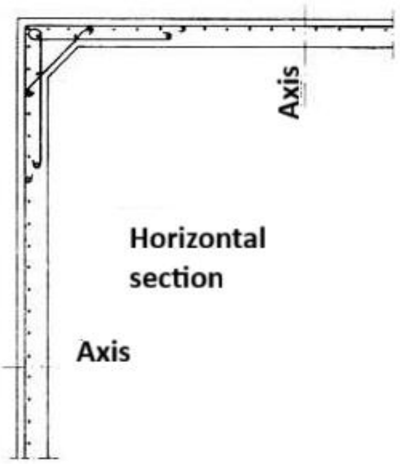


Fig. 3.13 **Fig. 3.14** **Fig. 3.15**

8. Covered tank with small horizontal surface but large height

Figure 3.16 shows the reinforcement layout in a horizontal section at an arbitrary level, far from the base slab. Near the base slab, vertical reinforcement is provided as shown in figure **3.17.**



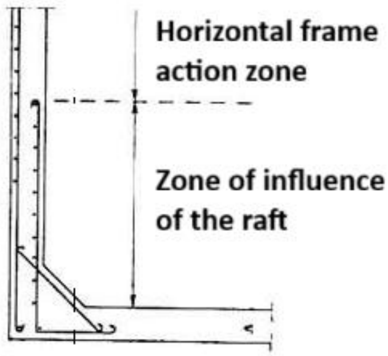


Fig. 3.16 **Fig. 3.17**

9. Covered tank with vertical or horizontal ribs

The reinforcement of the slab is arranged like that of a rectangular plate, as for the walls in the previous cases. The ribs act as flexural beams and are reinforced accordingly.

10. Uncovered tank with vertical ribs

The only specific feature of the reinforcement is the careful anchoring of the ribs into the base slab; this will be discussed further when studying base slabs.

11. Curved walls between ribs

Theoretically, no reinforcement is required since the vaults are generally under compression. However, a welded mesh or light reinforcement grid may be provided.

2.2. Circular water tank placed on the ground

In this case, the walls are in tension. The tension load (T) is given as follows:

$$T = \rho_w h. D/2 \tag{3.10}$$

Where, ρ_w = liquid density, h = tank height, D = Tank diameter.

- Cover:

The cover is considered as an RC dome.

- Raft foundations:

The total full tank weight must be taken into consideration such as:

- The cover weight and the live load on it.
- The weight of walls.
- The weight of the raft foundation.
- The weight of the contained liquid.

2.3.Elevated rectangular water tank

2.3.1. Cover

The cover can be considered as an RC slab or a dome according to their shape.

2.3.2. Bottom

The bottom is a simple slab supporting its own weight and the weight of the contained water. It also balances the bending moments at the ends introduced by the walls. There is no difficulty in the calculation. It should be noted that, since the walls provide support for the perimeter of this bottom slab, its reinforcement bars must be anchored vertically into the walls over the anchorage length (**Fig. 3. 18**).

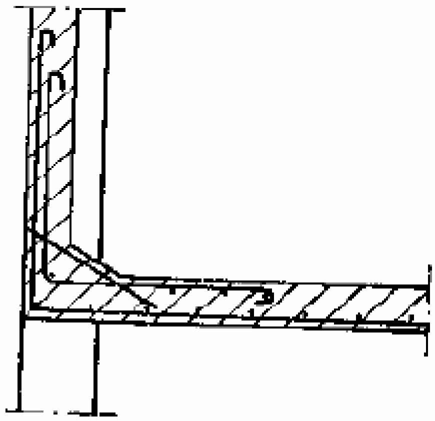


Fig. 3.18

2.3.3. Walls

In addition to their role in resisting water pressure, the walls transfer to the corner columns the reactions from the tank bottom. They therefore act as shear walls and must be calculated as such.

2.3.4. Support columns

The tank is supported by four corner columns, either projecting inside or outside the tank walls, or aligned with the mid-fibers of these walls.

They are subjected to combined bending, since in addition to the axial load, they must also resist wind effects. When the wind blows on the face of length b (**Fig. 3.19**), let V be the total wind load on the surface b*h:

$$V = \upsilon\, b\, h \tag{3.11}$$

Where, υ is the wind pressure per square meter.

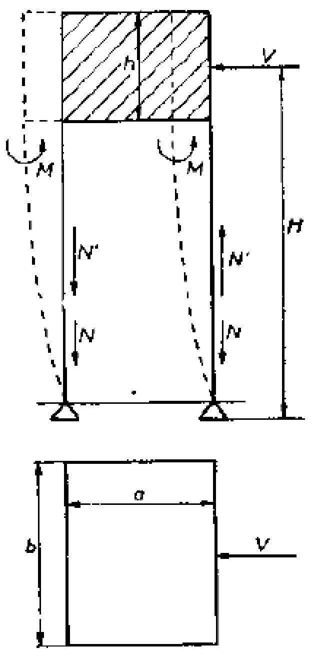


Fig. 3.19

The two columns 1 and 2 supporting the wall of height h, therefore of very high stiffness, each balance V/2. Assuming that the footings are hinged to the ground, the bending moment at the column head is:

$$M = \frac{1}{2} \times \frac{V}{2} \times H = \frac{VH}{4} \quad (3.12)$$

Furthermore, the positive and negative additional axial force introduced in the columns is:

$$N' = \frac{VH}{2} \times \frac{1}{a} = \frac{VH}{2a} \quad (3.13)$$

If N is the compressive force of the column due to the self-weight of the empty tank, the following condition must be satisfied N' < N.

Moreover, the column must be designed as being subjected to the axial force N (full tank) and to the bending moment M as given above.

If the water tower has a very great height, the moment *M* thus calculated can be very large. It can be divided by 2 if the columns are connected by four ground beams capable of resisting a moment *M/2* (**Fig. 3.20a**). Another option is to found the four columns on a general raft foundation which, for the columns, serves the same role as the ground beams. The columns may also be connected by intermediate horizontal ties (**Fig. 3.20b**), working together with the columns as vertical frame beams and calculated as such. Another solution consists of designing the columns in the form of angles (**Fig. 3.20c**) aligned with the walls.

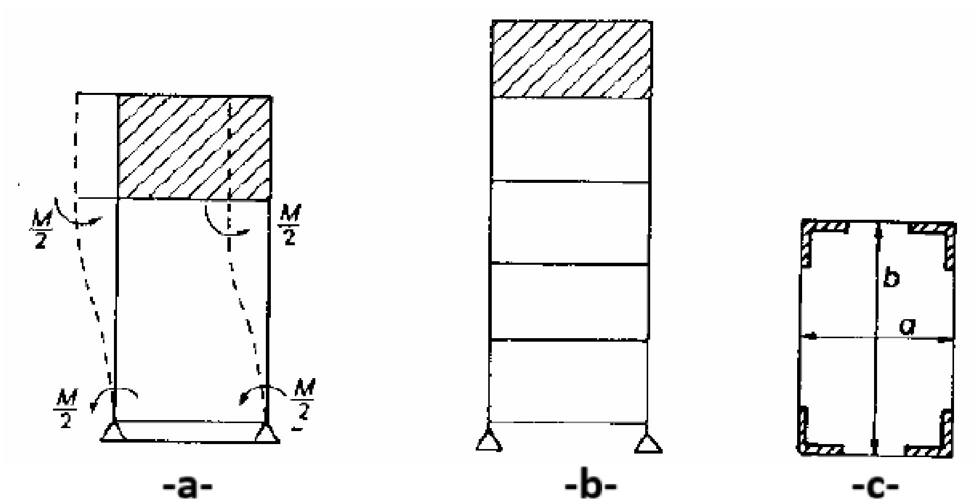


Fig. 3.20

2.3.5. Foundation

Formed by independent footings, by strip footings, or by a raft foundation, it presents no particular feature. In the case of isolated footings, it is advisable to provide flexible tie beams at the base of the columns.

2.4.Elevated circular water tank

2.4.1. Cover

As with circular reservoirs placed on the ground, the roof may be:

- a dome (parabolic or spherical),
- a solid circular slab,
- a ribbed slab,
- a central slab and an annular slab,
- a central dome and an annular slab,
- a central dome and a torus,
- a cone,
- a truncated cone,
- a slab floor.

2.4.2. *Vertical walls*

The calculation is again identical to that of the walls of tanks placed on the ground. At the lower part, the wall is always fixed, either on the bottom or on a truncated cone by means of a horizontal belt (**Fig. 3.21a**).

There is therefore a clamping effect. It can be assumed that it is identical to that of tanks placed on the ground. The walls may be straight, vertical, or inclined; we have already seen the calculation methods. They may also have arbitrary generatrixes (**Fig. 3.21b**).

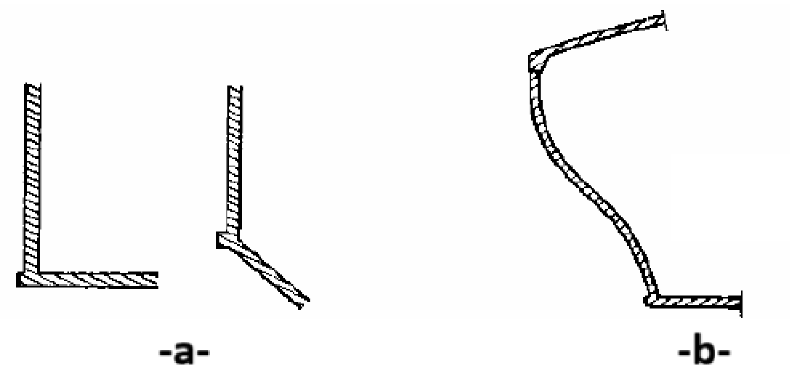


Fig. 3.21

The problem can also be solved step by step from the top, by treating each small vertical element of height Δz as a truncated cone (**Fig. 3.22**). We have:

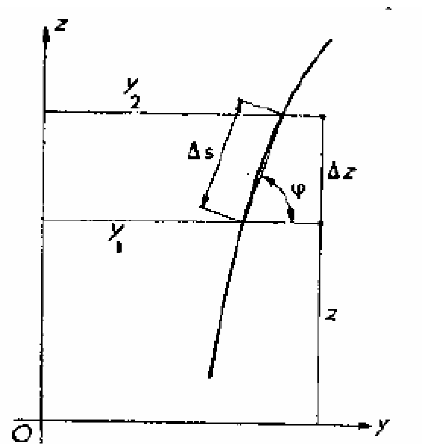


Fig. 3.22

$$tg\ \varphi = \frac{\Delta_z}{y_2 - y_1} \quad (3.14)$$

$$\Delta_s = \frac{\Delta_z}{sin\varphi} \quad (3.15)$$

- Self-weight (**Fig. 3.23a**):

$$n = \frac{p}{tg\,\varphi}\ ;\ t = \frac{p}{sin\,\varphi} \tag{3.16}$$

- Water (**Fig. 3.23b**):

$$n' = \frac{p'}{sin\,\varphi}\ ;\ t' = \frac{p\prime}{tg\,\varphi} \tag{3.17}$$

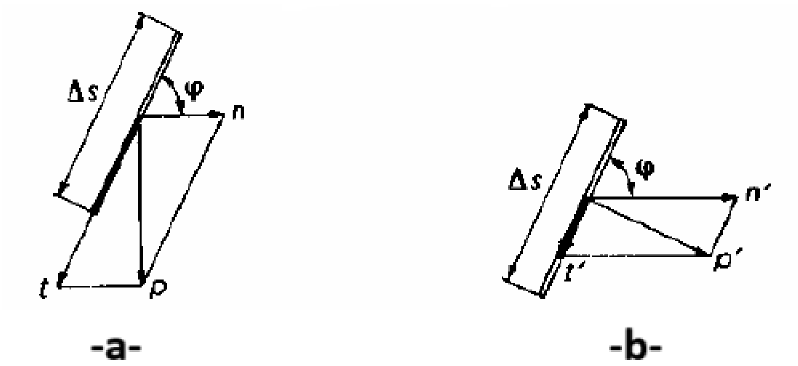


Fig. 3.23

p' is the thrust on the element Δs at the elevation: z + Δs/2.

We can then calculate by recurrence (**Fig. 3.24**) as follows:

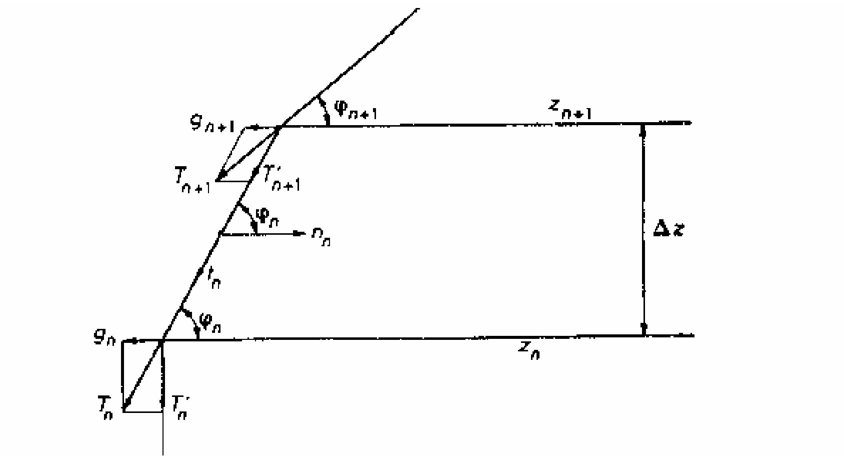


Fig. 3.24

t_n: the partial force on each element of index n along the generatrices;

n_n: the partial horizontal force on each element of index n;

T_n: the resultant force on the element of index n (taking into account everything above);

T'_n: the projection of T_n along the generatrices of the lower element of rank n-1;

g_n: the corresponding horizontal projection.

We can write:

$$\frac{T'_{n+1}}{\sin\varphi_{n+1}} = \frac{T_{n+1}}{\sin\varphi_n} \tag{3.18}$$

$$g_{n+1} = \frac{\sin(\varphi_n - \varphi_{n+1})}{\sin\varphi_n} T_{n+1} \tag{3.19}$$

The element of index *n* is therefore subjected to:

- a radial horizontal force:

$$N_n = n_n \frac{g_n + g_{n+1}}{2} \tag{3.20}$$

- a tangential force along the generatrices:

$$T_n = t_n + T'_{n+1} \tag{3.21}$$

2.4.3. Corbel

The corbel forms the junction between the walls of the tank, the tower, and the bottom of the tank. It can be either truncated-conical or toroidal.

The following equation expresses the meridional tensile force T in a truncated conical shell subjected to hydrostatic pressure and self-weight:

$$T = \frac{R^2 - r^2}{\sin^2\alpha}\,(\omega Z + \delta\, e\cos\,\alpha) - \frac{R^3 - r^3}{3\cos^2\alpha}\,\omega \tag{3.22}$$

- T: Meridional tensile force transmitted at the circular section of radius $r$by the conical shell portion between the upper radius $R$and the lower radius r. To obtain the membrane force per meter of generator, use $N_\phi = \frac{T}{2\pi r}$[kN/m].
- R: radius at the upper end of the truncated cone.
- r: radius at the considered section (lower end).
- α: angle of the cone generator with the horizontal (if measured from the vertical, replace by $90° - \alpha$).

- ω: specific weight of the liquid (ρg) [kN/m³].
- Z: depth of the upper generator (at R) below the free surface (height of water above R).
- δ: unit weight of concrete [kN/m³].
- e: thickness of the conical shell [m].

◆ The first term represents the combined hydrostatic pressure effect and the self-weight projection of the shell along its generator.

$$\frac{R^2-r^2}{\sin^2\alpha}(\omega Z+\delta e\cos\alpha) \tag{3.23}$$

◆ The second term accounts for the increase of pressure with depth (integration of ωzover the frustum) and is subtracted from the first.

$$\frac{R^3-r^3}{3\cos^2\alpha}\omega \tag{3.24}$$

➢ Including Self-Weight in the Bending Moment Calculation

Assumptions and Notations

- Generator length of the conical corbel: L(from top to bottom).
- Cone angle with the horizontal: α.
- Depth at the start of the corbel: z_0.
- Depth variation: $z(s)=z_0+s\sin\alpha$.
- Unit weight of water: ω(kN/m³).
- Unit weight of concrete: δ(kN/m³).
- Thickness of the corbel: e(m).

Distributed Loads (per meter of circumference)

- Hydrostatic pressure (variable with depth):

$$q_h(s)=\omega[h_{\text{water}}-z(s)]=\underbrace{\omega(h_{\text{water}}-z_0)}_{q_0}-\underbrace{\omega\sin\alpha s}_{k} \tag{3.25}$$

- Self-weight of the concrete (constant along the generator):

$$q_g = \delta e\cos\ \alpha \tag{3.26}$$

δ : concrete weight

e : concrete thickness

Bending Moment at the Base

$$M_{\text{base}} = \frac{\omega(h_{\text{water}}-z_0)L^2}{2} - \frac{\omega\sin\ \alpha L^3}{6} + \frac{\delta e\cos\ \alpha L^2}{2} \tag{3.27}$$

- The second term, $-\frac{\omega\sin\ \alpha L^3}{6}$, represents the variation of pressure with depth.
- The third term, $+\frac{\delta e\cos\ \alpha L^2}{2}$, is the contribution of the self-weight of the conical shell.

Geometric Alternative (using radii instead of L)

If $R$is the upper radius and $r$is the lower radius (at the base), for a cone:

$$L = \frac{R-r}{\cos\alpha} \tag{3.28}$$

Substitute this expression for Linto the equation above if you prefer to express the bending moment in terms of $R$and r.

2.4.4. Bottom

The base may consist of a thick or ribbed slab, a slab floor, or a mushroom slab. The base can also be spherical, conical, or toroidal. We have previously given the formulas for self-weight; the following expressions correspond to the water load.

2.4.4.1.Spherical Dome

According to **Fig. 3.25**, we have:

$$N_1 = R[\frac{H}{2} + \frac{R}{3} \cdot \frac{1+\sin\ \alpha+\sin^2\alpha}{1+\sin\ \alpha}] \tag{3.29}$$

$$N_2 = R[\frac{H}{2} + \frac{R}{3} \cdot \frac{1-2\sin\ \alpha-2\sin^2\alpha}{1+\sin\ \alpha}] \tag{3.30}$$

$$T_0 = R^2\sin\ \alpha_0\cos\ \alpha_0[\frac{H}{2} + \frac{R}{3} \cdot \frac{1+\sin\ \alpha_0+\sin^2\alpha_0}{1+\sin\ \alpha_0}] \tag{3.31}$$

If the dome is incomplete (**see Fig. 3.26**):

$$N_1 = \frac{R}{\cos^2\alpha}\left[\frac{H}{2}(\sin^2\alpha_1 - \sin^2\alpha) - \frac{R}{3}(\sin^3\alpha_1 - \sin^3\alpha)\right] \quad (3.32)$$

$$N_2 = \frac{R}{\cos^2\alpha}\left[(H - R\sin\alpha)\cos^2\alpha - \frac{H}{2}(\sin^2\alpha_1 - \sin^2\alpha) + \frac{R}{3}(\sin^3\alpha_1 - \sin^3\alpha)\right] \quad (3.33)$$

$$T_0 = R^2\sin\alpha_0\cos\alpha_0\left[\frac{H}{2}(\sin^2\alpha_1 - \sin^2\alpha_0) - \frac{R}{3}(\sin^3\alpha_1 - \sin^3\alpha_0)\right] \quad (3.34)$$

Where:

- R: radius of curvature of the spherical dome
- H: hydrostatic pressure at the crown (head of water)
- $\alpha, \alpha_0, \alpha_1$: angles measured from the vertical axis
- N_1, N_2: membrane forces in the meridional and hoop directions respectively
- T_0: total resultant of the meridional force

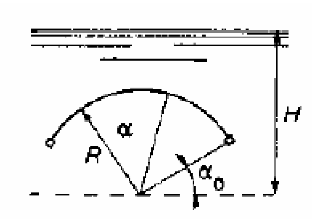


Fig. 3.25

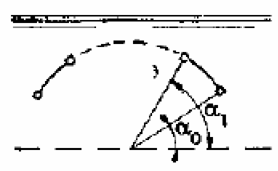


Fig. 3.26

2.4.4.2.Conical Roof (**Fig. 3.27**)

For a conical cover under hydrostatic pressure:

$$N_1 = \frac{x}{2}\tan\beta\left(H + \frac{2}{3}x\cos\beta\right) \quad (3.35)$$

$$N_2 = x\tan\beta(H + x\cos\beta) \quad (3.36)$$

$$T_0 = \frac{x_0^2\sin^3\beta}{2\cos\beta}\left(H + \frac{2}{3}x_0\cos\beta\right) \quad (3.37)$$

Where:

- x: distance measured along the cone generator from the apex,

- β: angle between the generator and the vertical,
- H: hydrostatic head at the upper edge,
- x_0: lower limit of the conical surface (where the reaction is taken),
- T_0: total resultant force in the meridional direction.

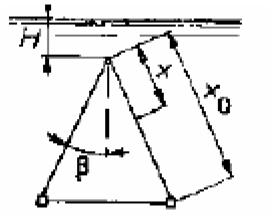


Fig. 3.27

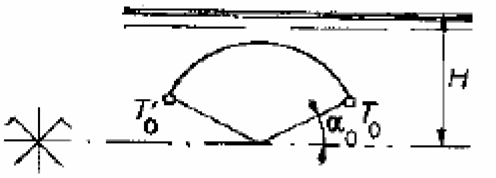


Fig. 3.28

2.4.4.3.Toroidal Roof (**Fig. 3.28**)

For a toroidal shell under hydrostatic pressure:

$$N_1 = \frac{r}{(R+r\cos\alpha)\cos\alpha}\left[RH\cos\alpha + \frac{rH}{2}\cos^2\alpha - \frac{rR}{2}\sin\alpha\cos\alpha - \frac{rR}{2}\left(\frac{\pi}{2}-\alpha\right) - \frac{r^2}{3}\left(1-\sin^3\alpha\right)\right]$$
(3.38)

$$N_2 = \frac{1}{\cos 2\alpha}\left[-\frac{rR}{2}\sin\alpha\cos\alpha + \frac{rH}{2}\cos^2\alpha - r^2\sin\alpha\cos\alpha + \frac{rR}{2}\left(\frac{\pi}{2}-\alpha\right) + \frac{r^2}{3}\left(1-\sin^3\alpha\right)\right]$$
(3.39)

$$T_0 = r^2\sin\alpha_0\cos\alpha_0\left[RH\cos\alpha_0 + \frac{rR}{2}\cos^2\alpha_0 - rR\sin\alpha_0\cos\alpha_0 - \frac{rR}{2}\left(\frac{\pi}{2}-\alpha_0\right) - \frac{r^2}{3}\left(1-\sin^3\alpha_0\right)\right]$$ (3.40)

2.4.5. Support columns

2.4.5.1.Loads to Be Considered

The loads to be taken into account are:

- The self-weight,
- The water pressure from the reservoir,
- The transverse wind pressure.

These effects have already been analyzed in the *previous* section.

2.4.5.2. Cylindrical Thin-Shell Tower

The self-weight can be studied without difficulty, as can the effect of the water contained in the reservoir. Under these two effects, the tower is uniformly compressed, and therefore no special computational considerations are required. The wind introduces additional forces that can be easily calculated according to design codes (**see Fig. 3.29**).

Let:

- F_1be the wind force acting on the reservoir,
- F_2be the wind force acting on the cylindrical tower.

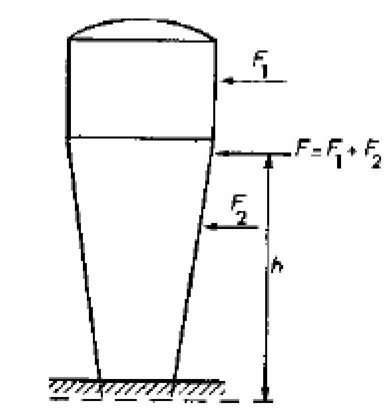


Fig. 3.29

The resultant force is:

$$F = F_1 + F_2 \tag{3.41}$$

acting at a height $h$above the foundation level.

If Nrepresents the total axial load, we have:

$$N = N_1 + N_2 \tag{3.42}$$

where:

- N_1: self-weight of the structure,
- N_2: weight of the water in the reservoir.

- Bending Moment: The moment due to wind is:

$$M = Fh \tag{3.43}$$

- Possible Load Cases: Four situations are to be considered:

1. Empty tank, no wind:

$$N = N_1; M = 0 \tag{3.44}$$

2. Empty tank, with wind:

$$N = N_1; M = Fh \tag{3.45}$$

3. Full tank, no wind:

$$N = N_1 + N_2; M = 0 \tag{3.46}$$

4. Full tank, with wind:

$$N = N_1 + N_2; M = Fh \tag{3.47}$$

All four cases must be considered both in terms of stresses and stability.

- Stresses

In general, case 4 gives the maximum stress in the tower shell.

$$\sigma = \frac{N}{S} + \frac{My}{I} \tag{3.48}$$

where:

- σ: total normal stress in the shell,
- N: axial load (self-weight + water weight),
- M: bending moment due to wind,
- y: distance from the neutral axis.

- Section Properties

$$S = \pi De + nA \tag{3.49}$$

$$I = \frac{\pi De^3}{4} + nA\frac{D^2}{4} \tag{3.50}$$

with:

- D: mean diameter of the cylindrical shell,
- e: wall thickness,
- A: section of vertical reinforcement,
- n: number of stiffeners or reinforcement bands.

2.4.5.3.Pylon on Vertical Columns Without Cross-Braces

The columns are generally anchored on the lower ring beam of the tank (see Fig. 3-236). This ring beam is subjected to bending moments and torsional moments (in addition to the shear force, which is easy to compute).

The magnitude of these internal forces depends on the number of columns supporting the tank.

The stability condition is the same as that of the tower. The forces due to vertical loads are easy to calculate. The wind produces two main effects on the columns:

- Vertical loads, either downward or upward,
- Bending moments.

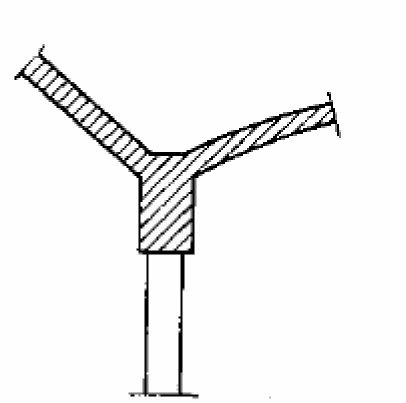


Fig. 3.30

2.4.5.4.Pylons on Vertical Columns with Cross-Braces

The stability condition is the same as that for pylons without cross-braces. Similarly, the additional loads due to wind remain the same. Only the bending effects differ, due to the horizontal deformation caused by the wind.

Figure 3.31 shows three possible bracing configurations. The columns, reservoir, bracings, and foundation together form a single calculable structural unit at full scale. For example, in **Figure 3.32**, a system of six columns with a single intermediate bracing ring is shown.

In this case, the wind load is shared and balanced between the two braced elements, $E_1$and E_2.

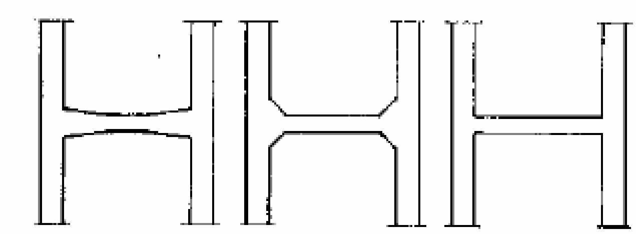


Fig. 3.31

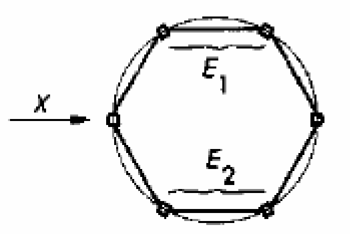


Fig. 3.32

2.4.5.5.Pylon Tower Stiffened by Buttresses (counterfort)

The general stability condition under wind loading is always the same: the resultant must pass through the central core of the structure.

From a stress point of view—and except for exceptional cases—the behavior remains identical to that of the other configurations described previously.

A thin shell by itself would generally be sufficient. However, a combined bending analysis can always be performed on a homogenized section, assumed to be fully compressed.

When choosing a pylon system, the aesthetic aspect should play a major role, followed by the cost factor. From this standpoint, the simplest pylon design will, as always, be the most economical. **Figure 3.33** illustrates this.

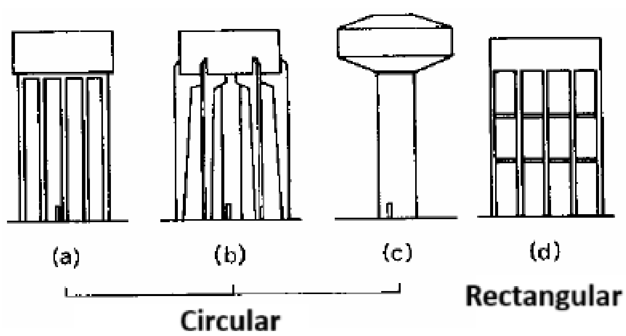


Fig. 3.33

2.4.6. Foundations

Foundations can be constructed as continuous footings, isolated footings, mat foundations, caissons, or pile foundations.

2.4.6.1. Continuous Footing

This is the most economical type for a tower with a thin shell, either stiffened or not by buttresses (**see Fig. 3.34**). The calculation can be made for a 1-meter-wide strip, similar to that of a straight continuous footing.

The only particular case to note concerns a tower with an inclined wall (**Fig. 3.35**). The horizontal component of the inclined load at the footing level introduces a tangential stress that either compresses or tensions the circular footing.

In the case of tension, circular reinforcement balances this stress. It is also possible to construct a continuous footing under a pylon with columns. This footing may be circular or polygonal in shape. It is analyzed for bending and torsion in the same way as the top ring beam (**Fig. 3.36**).

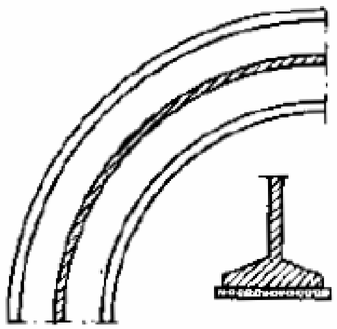


Fig. 3.34

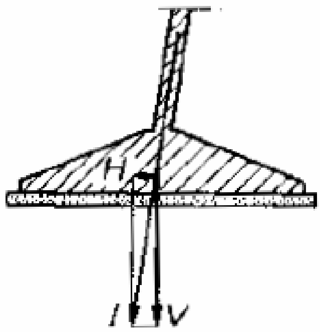


Fig. 3.35

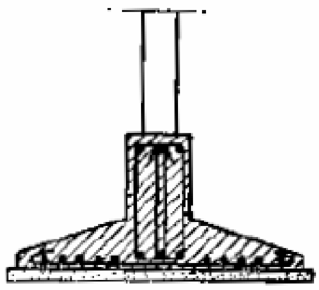


Fig. 3.36

2.4.6.2. Isolated Footings

Isolated footings are provided under the pylon columns; in such cases, this is the most economical solution. It is advisable to connect the footings with a polygonal tie beam (**Fig. 3.37**), which will be in tension or compression if the pylon columns are inclined (**Fig. 3.38**).

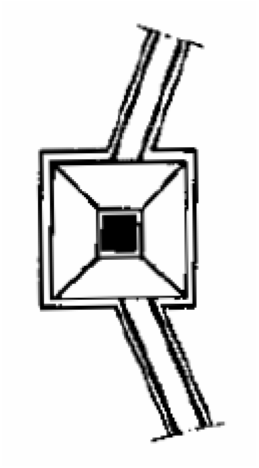


Fig. 3.37

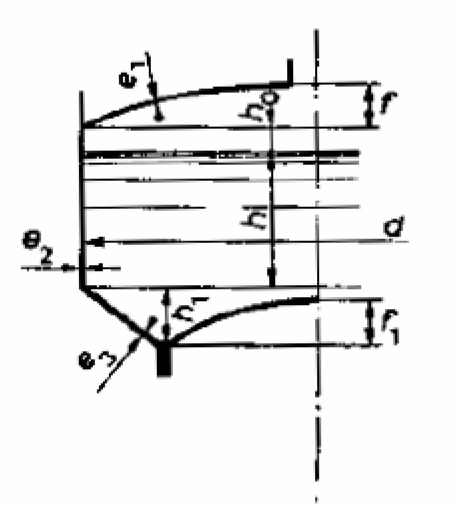


Fig. 3.38

2.4.6.3.Pylon on raft Foundation

This solution is recommended for poor-quality soil. The mat (raft) is designed in the same way as for the bottom slabs of circular reservoirs resting on the ground. For a pylon supported by columns, it is recommended to stiffen the mat with a peripheral beam designed for bending.

2.4.6.4. Foundation on Caissons

The caissons are placed under the columns or distributed around the periphery of the tower with a solid wall shell. In both cases, the columns and wall are provided with a reinforced-concrete footing (either isolated or continuous) resting on the caissons.

3. APPLICATION

Calculate a reservoir of 350 m³ mounted on a tower 17.00 m above the ground, divided into two compartments.

Weighted wind pressure per square meter of cylindrical or conical surface:

$200 \times 0.60 = 120$ daN/m^2.

Allowable bearing pressure on the ground: 2 bar (see **Fig. 3.39**).

Allowable stress for mild steel:

- Foundation and tower: 13 hbar,
- Tank (waterproofing): 10 hbar.

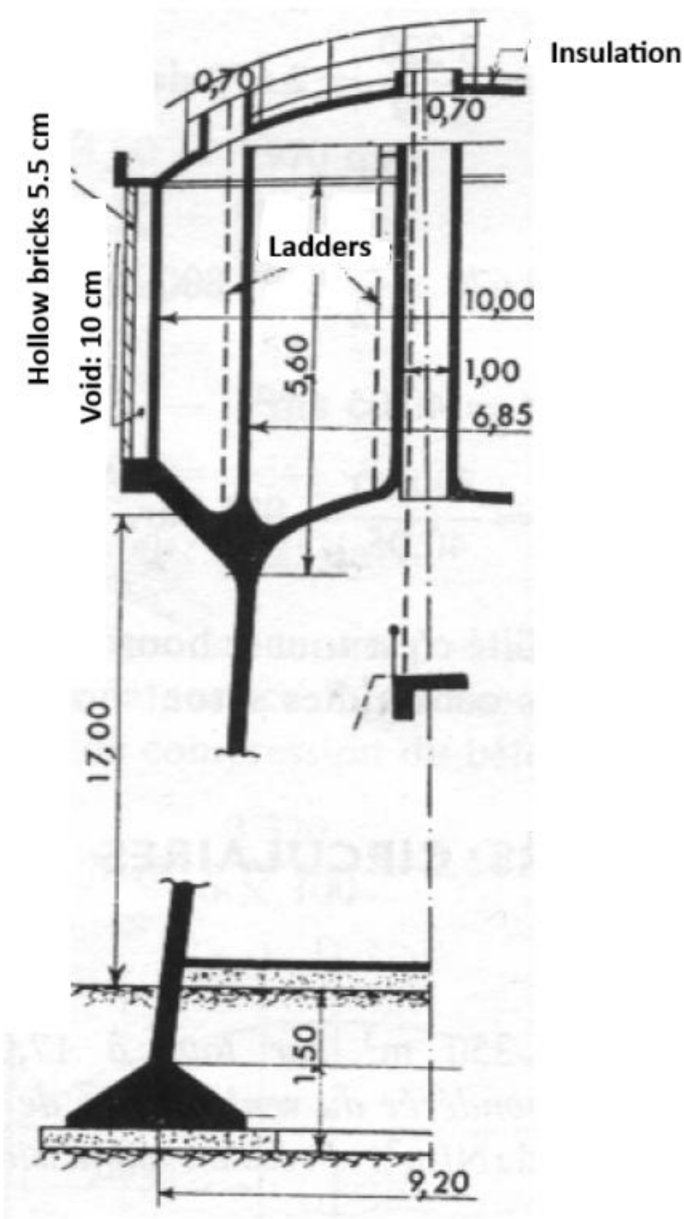


Fig. 3.39

3.1. External Cylindrical Wall

- Water height: $H = 4.00$ m
- Internal diameter: $D = 10.00$ m
- Wall thickness: $t = 7$to 11 cm (info only)
- Allowable steel stress used in the example: $\sigma_a = 1000$ daN/cm^2

1) Water thrust per meter height

$$Q = \frac{1000\,H^2}{2} = 500H^2$$

At $H = 4$m:

$$Q = 500 \times 4^2 = 500 \times 16 = 8000 \text{ daN/m}$$

2) Weighted membrane tension in the wall

$$T = \frac{1.2\,Q\,D}{2} = 300H^2D$$

At $H = 4$m and $D = 10$m:

$$T = 300 \times 4^2 \times 10 = 300 \times 16 \times 10 = 48{,}000 \text{ daN/m}$$

3) Required steel area

$$A = \frac{T}{\sigma_a} = \frac{300H^2D}{\sigma_a}$$

Numerically (with $\sigma_a = 1000$ daN/cm^2):

$$A = \frac{48{,}000}{1000} = 48 \text{ cm}^2$$

So, the maximum steel area at the base (where $H = 4$m) is:

$$\boxed{A_{\text{base}} = 48 \text{ cm}^2 \text{ (per meter height)}}$$

4) Distribution along the height (parabolic law)

From the page:

$$A(H) = \frac{300\,H^2\,D}{\sigma_a}$$

With $D = 10$m and $\sigma_a = 1000$ daN/cm^2:

$$\boxed{A(H) = 3\,H^2 \text{ cm}^2 \text{ per meter height}}$$

- Top ($H = 0$): $A = 0$
- Mid-height ($H = 2$): $A = 3 \times 2^2 = 12$ cm^2
- Base ($H = 4$): $A = 3 \times 4^2 = 48$ cm^2(as found)

You "plot Avs H" and place the ring bars (circumferential steel) accordingly, starting from the base.

5) Note from the text on vertical (generator) bars

Meaning: provide Ø6 mm vertical bars, about 320 around the circumference, with doubling around mid-height (for crack control).

Check spacing: circumference $= \pi D \approx 31.42$ m→ spacing $s \approx 31.42/320 \approx 0.098$ m≈ 10 cm (reasonable).

3.2.Wall Ring (Truncated Cone Inclined at 45°)

1) Total load at the base of the cylindrical wall:

Dome and accessories: $37\,800 + 1\,200 = 39\,000$ daN

Cornice (ring beam): $\pi \times 10.49 \times 2\,500 \times 0.025 = 2\,060$ daN

Cylindrical wall: $\pi \times 10.13 \times 4.00 \times 2\,500 \times 0.08 = 25\,300$ daN

Ring: $\pi \times 10.37 \times 2\,500 \times 0.045 = 3\,600$ daN

Weight of insulation and external coating: $\pi \times 10.48 \times 3.83 \times 130 = 16\,400$ daN

Hence, the total load is:

$$Q = 86\,420 \text{ daN} \approx 86\,500 \text{ daN}$$

This total load $Q$is decomposed into a compression force Q_2in the truncated cone and a tension force Q_1in the ring, which balances the compression.

$$Q_1 = Q = 86\,500 \text{ daN}$$

2) Tensile force in the ring:

$$T = \frac{Q}{\pi d} \times \frac{d}{2} = \frac{Q}{2\pi}$$

Substituting numerical values:

$$T = \frac{86\,500}{2\pi} = 13\,750 \text{ daN}$$

3) Required steel section:

$$A = \frac{T}{\sigma_a} = \frac{13\,750}{1\,000} = 13.75 \text{ cm}^2$$

Therefore, the reinforcement to be provided is: **4Φ20**

3.3.Truncated Cone

Thickness: 14 cm

Smaller base diameter: 6.975 m

Larger base diameter: 10.150 m

Mean diameter: $D_m = 8.565$ m

Height: 1.59 m

Slant height: $L = 1.59\sqrt{2} = 2.25$ m

Weight per square meter

Self-weight:

$$2\,500 \times 0.14 = 350 \text{ daN/m}^2$$

Coating:

$$50 \text{ daN/m}^2$$

Total:

$$p = 400 \text{ daN/m}^2$$

This load can be replaced in the calculations by an **equivalent fictitious water height** h, such that:

$$1\,000h = p\cos\ 45°$$

Hence:

$$h = \frac{p\cos\ 45°}{1\,000} = \frac{400}{1\,000\sqrt{2}} = 0.28 \text{ m}$$

Average water height on the truncated cone

$$H = 4.70 + 0.28 = 4.98 \text{ m}$$

Because of the **1.20 load factor** applied to the water surcharge, the calculation is carried out using a **fictitious height**:

$$H' = 4.7 \times 1.2 + 0.28 = 5.92 \text{ m}$$

Average normal pressure

$$P_m = 1\,000H' = 1\,000 \times 5.92 = 5\,920 \text{ daN/m}^2$$

Horizontal component:

$$P = 1\,000H'\sqrt{2} = 1\,000 \times 5.92 \times 1.414 = 8\,370 \text{ daN/m}^2$$

Total tension along the slant height of the cone

$$T = 1\,000H'\sqrt{2}L\frac{D_m}{2}$$

Substituting the values:

$$T = 1\,000 \times 5.92\sqrt{2} \times 2.25 \times \frac{8.565}{2} = 81\,000 \text{ daN}$$

Total steel section required

$$A = \frac{T}{\sigma_a} = \frac{81\,000}{1\,200} = 67.5 \text{ cm}^2$$

Hence, the reinforcement required is:

22 bars of 20 mm diameter

Total load at the base of the cone

Load at the base of the cylindrical wall: $86\,420$ daN

Cone and coating: $\pi \times 8.565 \times 2.25 \times 450 = 27\,200$ daN

Water above the truncated cone (factored):

$$\frac{\pi}{4}(10.00^2 - 7.10^2) \times 1\,000 \times 4.70 \times 1.2 = 218\,000 \text{ daN}$$

Total:

$$Q = 331\,620 \text{ daN} \approx 332\ kdaN$$

Steel section along the generatrices

$$A = \frac{332\,000}{\frac{4}{5} \times 1\,200} = 346 \text{ cm}^2$$

Reinforcement adopted: 66Φ20 + 66Φ16

Concrete compression per linear meter

$$N'_b = \frac{P}{\sin\ 45° \cdot \pi d} = \frac{331\,620 \times \sqrt{2}}{\pi \times 6.975} = 21\,600 \text{ daN/m}$$

Therefore:

$$\sigma'_b = \frac{21\,600}{100 \times 14} = 15.4 \text{ bar}$$

3.4. Bottom Dome

(Spherical, lowered by 1/8, thickness 13 cm)

Surface area of the dome

$$S = 2 \times 3.14 \times \frac{3.48^2 + 0.87^2}{2 \times 0.97} = 39 \text{ m}^2$$

Load per square meter

Self-weight:

$$2\,500 \times 0.13 = 325 \text{ daN/m}^2$$

Coating:

$$+40 \text{ daN/m}^2$$

Total:

$$p = 365 \text{ daN/m}^2$$

Weight of the dome

$$365 \times 39 = 14\,300 \text{ daN}$$

Weighted water load above the dome

$$\frac{\pi}{4}(6.85^2 - 1.18^2) \times 1\,000 \times 5.00 \times 1.2 = 218\,000 \text{ daN}$$

Interior chimneys

$$\pi \times 1.08 \times 4.80 \times 250 = 3\,500 \text{ daN}$$

Total load

$$Q = 235\,800 \text{ daN} \approx 236\ kdaN$$

Steel section (meridional reinforcement)

$$A' = \frac{236\,000}{\frac{4}{5} \times 1\,200} = 246 \text{ cm}^2$$

Provide:

105 bars of 14 mm and 105 bars of 10 mm

Compression at the base of the dome

$$N = \frac{23\,600}{39} \times \frac{(3.48^2 + 0.87^2)^2}{4 \times 3.48^2 \times 0.87} = 23\,700 \text{ daN}$$

Therefore, the **compressive stress** at the base is:

$$\sigma_b' = \frac{23\,700}{100 \times 13} = 18 \text{ bar}$$

3.5. Base Ring

(Outer tank empty, inner tank full)

Tension produced by the bottom dome

$$T = \frac{236\,000}{\pi} = 75\,000 \text{ daN}$$

Total load on the circumference at the cone base

$$Q = 332\,000 - 218\,000 = 114\,000 \text{ daN}$$

Compression in the ring

$$\frac{114\,000}{2\pi} = 18\,200 \text{ daN}$$

Resulting tension in the ring

$$T' = 75\,000 - 18\,200 = 56\,800 \text{ daN}$$

Required steel area

$$A = \frac{56\,800}{1\,200} = 47.3 \text{ cm}^2$$

Hence, the required reinforcement is: 2 x 5Φ25 = 50cm²

3.6.Inner Cylindrical Wall

Thickness: from 0.06 m to 0.08 m

Internal diameter: 6.85 m

Water height: 5.30 m

Steel area of circular reinforcement to be distributed in the wall

$$A_t = \frac{T}{\sigma_a} = \frac{250 \times 1.2H^2D}{\sigma_a'}$$

Substituting the numerical values:

$$A_t = \frac{300 \times 5.30^2 \times 6.85}{1\,200} = 48.10 \text{ cm}^2$$

Distribution of reinforcement

The reinforcement is to be distributed along the **generatrices** (verticals):

220 bars of 6 mm diameter, with half interrupted at mid-height.

3.7. Supporting Tower

Thickness: 0.10 m

Reinforcement along generatrices: Ø8 spaced at 20 cm

Circular reinforcement: 5 Ø8 per meter

Total weight of the structure at the base of the tower

- Inner cylindrical wall:

$$\pi \times 6.970 \times 5.50 \times 295 = 35\,600 \text{ daN}$$

- Load transmitted by the cone:

$$332\,000 \text{ daN}$$

- Load transmitted by the bottom dome:

$$236\,000 \text{ daN}$$

- Weight of the tower with coatings:

$$124\,000 \text{ daN}$$

- Platforms and ladders:

$$4\,500 \text{ daN}$$

- Mass concrete footing:

$$\pi \times 9.20 \times 1.50 \times 0.10 \times 2\,000 = 8\,700 \text{ daN}$$

Total:

$$Q = 740\,800 \text{ daN} \approx 741 \text{ kdaN}$$

Effect of wind after weighting

- Tank:

$$10.54 \times 6.00 \times 120 = 7\,600 \text{m\textbackslash daN}$$

- Tower:

$$8.10 \times 16.45 \times 120 = 16\,000 \text{m\textbackslash daN}$$

Overturning moment:

$$M = 7\,600 \times 21.40 + 16\,000 \times 9.70 = 318\,000 \text{m\textbackslash daN}$$

Eccentricity of the resultant:

$$e = \frac{M}{N} = \frac{318\,000}{670\,000} = 0.475 \text{ m} < \frac{9.20}{8} = 1.15 \text{ m}$$

☑ **Stability is ensured.**

Moment of inertia of the foundation

$$I = \frac{\pi}{64}(10.7^4 - 7.7^4) = 470 \text{ m}^4$$

Maximum soil stress

$$\sigma_s = \frac{670\,000}{\pi \times 9.20 \times 10^4 \times 1.50} + \frac{318\,000 \times 5.35}{10^4 \times 470} = 1.445 + 0.365 = 1.8 \text{ bar} < 2 \text{ bar}$$

Moment of inertia of the tower at foundation level

$$I' = \frac{\pi}{64}(9.25^4 - 9.15^4) = 13.1 \text{ m}^4$$

Concrete stress

Compression:

$$\sigma_c = \frac{670\ 000}{\pi \times 9 \times 20 \times 10^4 \times 0.10} + \frac{318\ 000 \times 4.625}{10^4 \times 13.1} = 23.2 + 11.2 = 34.4 \text{ bar}$$

Tension:

$$\sigma_t = 23.2 - 11.2 = 12 \text{ bar}$$

The compressive stress is very acceptable.

The tensile stress is relatively small, and the concrete could withstand it if necessary.

The tower is practically reinforced longitudinally with Ø8 bars spaced every 20 cm.

For a more precise verification, M. Hahn's charts can be applied using the following data:

$$r = 460 \text{ cm}, e = 10 \text{ cm}, Q = 670 \text{ kdaN}, M = 318 \text{ mkdaN}, \sigma_b' = 60 \text{ bar}$$

Calculations

$$M = \frac{670\ 000}{2\pi \times 460 \times 10 \times 60} = 0.386$$

$$N = \frac{31\ 800\ 000}{2\pi \times 460^2 \times 10 \times 60} = 0.038$$

CHAPTER 04: RC SILOS

1. GENERALITIES

1.1.Definition

Silos are containers intended for storing bulk granular materials, such as agricultural products or raw materials, as well as materials intended for industry. They are made up of one or more cells of polygonal or circular section.

Put simply, a silo is a storage structure; anything stored in a silo is referred to as silage. What it looks like is largely dependent on what it holds and how it's unloaded.

Fig. 4.1. Tower silo or a grain elevator.

1.2.Classification

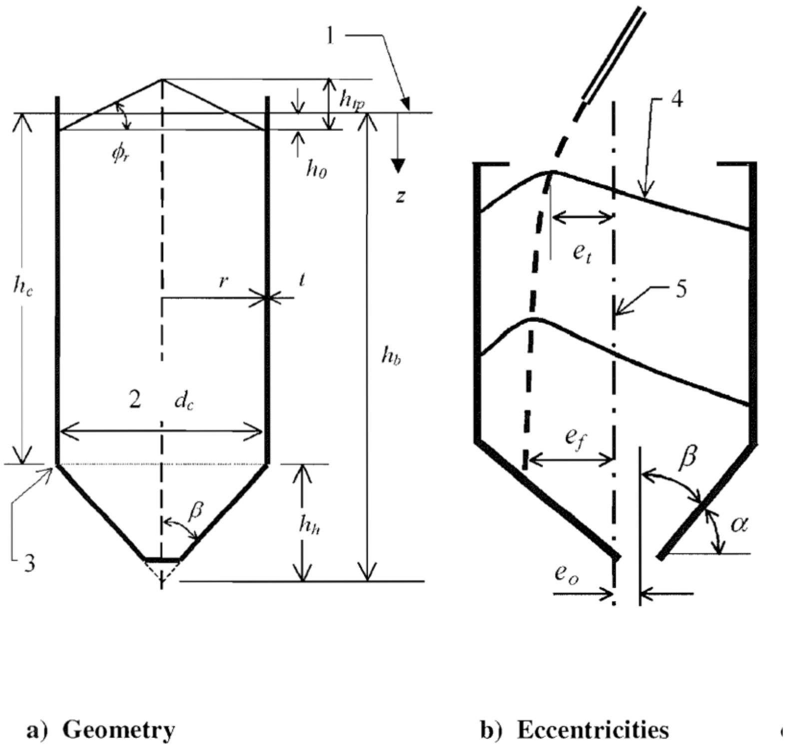


Fig. 4.2. Tower silo or a grain elevator.

1.2.1. According to slenderness of the silo

Table 4.1. Classification according to slenderness.

Type of silos	Description
Slender silos	$h_c / d_c \geq 2$
Intermediate slenderness silos	$1 < h_c / d_c < 2$
Squat (Thick) silos	$0.4 < h_c / d_c \leq 1$
Retaining silos	$h_c / d_c \leq 0.4$

1.2.2. According to action assessment

Table 4.2. Classification according to action assessment.

Class	Description
1	Silos with capacity below 100 tonnes.
2	All silos that are not in classes 1 and 3
3	- Silos with capacity in excess of 10 000 tonnes - Silos with capacity in excess of 1000 tonnes in which any of the following design situations occur: 1- eccentric discharge with $e_0 / d_c > 0.25$. 2- squat silos with top surface eccentricity with $e_t / d_c > 0.25$.

2. ACTIONS ON SILOS

Actions on silos shall be determined taking account of:

- The silo structure,
- The stored solid properties,
- the discharge flow patterns that arise during the process of emptying.

2.1.Solid properties

2.1.1. Bulk unit weight (γ)

The bulk unit weight should be determined at a particle packing density and at a stress level corresponding to the position in the stored solid in the silo where the maximum vertical stress after filling occurs.

2.1.2. Coefficient of wall friction μ

Tests to determine the wall friction coefficient or the calculation of loads should be determined at a particle packing density and at a stress level corresponding to the position in the stored solid in the silo where the maximum assessed horizontal filling pressure on the vertical wall after filling occurs. The upper and lower values of the wall friction coefficient are given as follows:

$$\mu_u = a_\mu \mu_m \quad (4.1)$$

$$\mu_l = \frac{\mu_m}{a_\mu} \tag{4.2}$$

Where a_μ and μ_m are the modification coefficient and the mean value of the wall friction coefficient, respectively.

2.1.3. Angle of internal friction Φ_i

The loading angle of internal friction (arctan of the ratio of shear stress to normal stress at failure during virgin loading) should be determined at a particle packing density and at a stress level corresponding to the position in the stored solid in the silo where the maximum vertical stress after filling occurs. The upper and lower values of angle of internal friction are given as follows:

$$\Phi_u = a_\Phi \Phi_m \tag{4.3}$$

$$\Phi_l = \frac{\Phi_m}{a_\Phi} \tag{4.4}$$

Where a_Φ and Φ_m are the modification coefficient and the mean value of the angle of internal friction, respectively.

2.1.4. Lateral pressure ratio K

The lateral pressure ratio *K* (ratio of mean horizontal to mean vertical pressure) should be determined at a particle packing density and at a stress level corresponding to the position in the stored solid in the silo where the maximum vertical stress after filling occurs. The upper and lower values of the lateral pressure ratio are given as follows:

$$K_u = a_k K_m \tag{4.5}$$

$$K_l = \frac{K_m}{a_k} \tag{4.6}$$

Where a_K and K_m are the modification coefficient and the mean value of the lateral pressure ratio, respectively.

2.1.5. Cohesion c

The cohesion *c* of the solid varies with the consolidating stress that has been applied to the solid. It should be determined at a particle packing density and at a stress level corresponding to the position in the stored solid in the silo where the maximum vertical stress occurs after filling.

2.1.6. Patch load solid reference factor C_{op}

In the case of asymmetrical filling or emptying, the C_{op} factor is used as a correction factor. This factor can equal to:

$$C_{op} = 3.5\, a_{\mu} + 2.5\, a_{k} - 6.2 \tag{4.7}$$

2.2. Loads on the vertical walls of silos

- Symmetrical loads: Symmetrical loads are fixed loads distributed uniformly along the circumference of the silo. Discharge loads result from increasing uniform loads by an increase factor.
- Patch load: In addition to fixed loads, additional free loads are generally applied to silos. The distribution of non-symmetrical loads in a silo is caused by actions due to imperfections or eccentricities during the filling and emptying of solids.

2.2.1. Slender silos

2.2.1.1.Filling loads

a. Symmetrical filling load

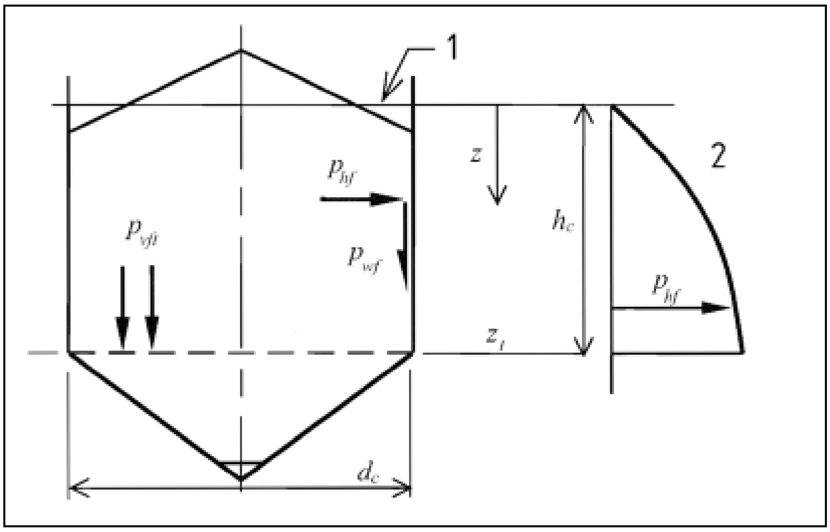


Fig. 4.3. Symmetrical filling pressures in the vertical-walled segment.

The values of horizontal pressure P_{hf} wall frictional traction P_{wf} and vertical pressure P_{vf} at any depth after filling and during storage should be determined as:

$$p_{hf}(z) = p_{h0} Y_J(z) \tag{4.8}$$

$$p_{wf}(z) = \mu\, p_{h0} Y_J(z) \tag{4.9}$$

$$p_{vf}(z) = \frac{p_{h0}}{K} Y_J(z) \tag{4.10}$$

In which:

$$p_{h0} = \gamma\, K\, z_0 \tag{4.11}$$

$$z_0 = \frac{1}{K\mu}\frac{A}{U} \tag{4.12}$$

$$Y_J(z) = 1 - e^{-z/z_0} \tag{4.13}$$

where:
γ is the characteristic value of the unit weight

μ is the characteristic value of the wall friction coefficient for solid sliding on the vertical wall

K is the characteristic value of the lateral pressure ratio

z is the depth below the equivalent surface of the solid

A is the plan cross-sectional area of the silo

U is the internal perimeter of the plan cross-section of the silo

The resulting characteristic value of the vertical force (compressive) in the wall n_{zSk} per unit length of perimeter after filling at any depth z should be determined as:

$$n_{zSK} = \int_0^{z} p_{wf}(z)\, dz = \mu\, p_{h0}\left[\, z - z_0\, Y_J(z)\right] \tag{4.14}$$

b. Filling path load

The filling patch load, or an appropriate alternative, shall be used to represent accidental asymmetries of loading associated with eccentricities and imperfections in the filling process.

- For silos in Action Assessment Class I, the filling patch load may be ignored.
- For silos used for the storage of powders that become aerated during the filling process, the filling patch load may be ignored.

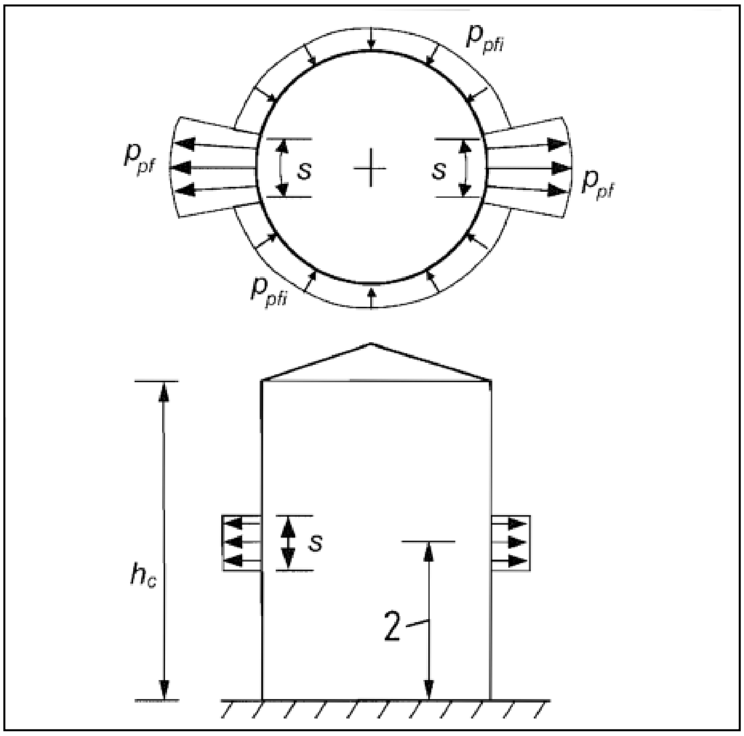


Fig. 4.4. Circular silos: side elevation and plan view of the filling patch load.

This filling patch pressure P_{pf} is used for the correction of non-symmetrical loads during filling. The reference magnitude of the filling patch pressure P_{pf} should be taken as:

$$p_{pf} = C_{pf}\, p_{hf} \tag{4.15}$$

In which:

$$C_{pf} = 0.21C_{op}[1 + 2E^2]\left(1 - e^{\{-1.5[(h_c/d_c)-1]\}}\right) \tag{4.16}$$

$$E = 2\, e_f /\, d_c \tag{4.17}$$

If the expression of C_{pf} is negative, C_{pf} should be taken equal 0.

where:

e_f is the maximum eccentricity of the surface pile during filling;

P_{hf} is the local value of the filling pressure at the height at which the patch load is applied;

C_{op} is the patch load solid reference factor for the solid.

- The height of the zone on which the patch load is applied (**see Figure 4.4**) should be taken as:

$$s = \pi\, d_c /16 \approx 0.2\, d_c \tag{4.18}$$

b.1. Filling patch load: thick-walled circular silos *($d_c/t < 200$)*

In addition to the outward patch pressure P_{pf} the remainder of the silo circumference over the same height of wall (**see Figure 4.5**) should be subjected to an inward patch pressure P_{pfi} given by:

$$p_{pfi} = p_{pf}/7 \tag{4.19}$$

b.2. Filling patch load: non-circular silos

The magnitude of the uniform symmetrical pressure increases on the non-circular wall $P_{pf,nc}$ should be taken as:

$$p_{pf,nc} = 0.36 p_{pf} \tag{4.20}$$

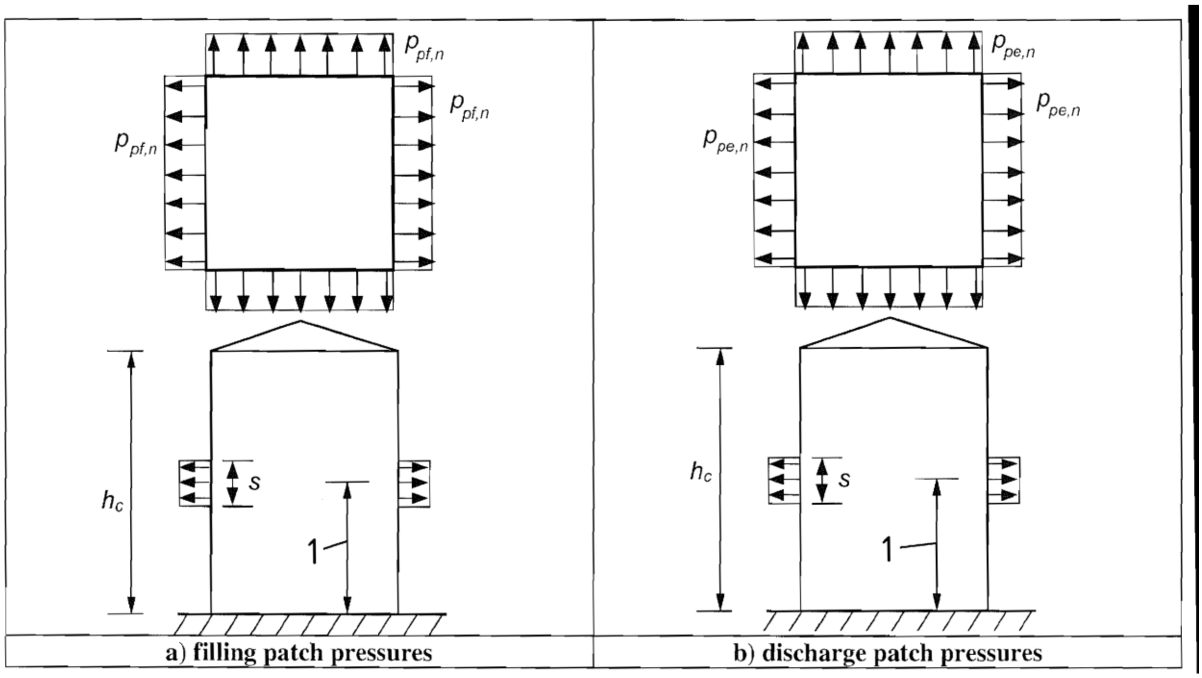


Fig. 4.5. Non-circular silos: side elevation and plan view of patch loads.

2.2.1.2.Discharge loads

a. Symmetrical discharge load

Symmetrical increases in the discharge load shall be used to represent the possible transitory increases in pressure that occur on silo walls during the discharge process.

For silos in all Action Assessment Classes, the symmetrical discharge pressures P_{he} and P_{we} should be determined as:

$$p_{he} = C_h \, p_{hf} \tag{4.21}$$

$$p_{we} = C_w\, p_{wf} \tag{4.22}$$

where:

C_h is the discharge factor for horizontal pressure

C_w is the discharge factor for wall frictional traction.

For silos in all Action Assessment Classes that are unloaded from the top (no flow within the stored solid), the values of C_h and C_w may be taken as: $C_h = C_w = 1$.

For slender silos in Action Assessment Classes 2 and 3, the discharge factors should be taken as: $C_h = 1.15$ et $C_w = 1.1$.

For slender silos in Action Assessment Class 1, where the mean value of the material properties K and μ have been used for design, the discharge factors should be taken as:

$$C_h = 1.15 + 1.5\,(1 + 0.4e/d_c)\, C_{op} \tag{4.23}$$

$$C_w = 1.4\,(1 + 0.4e/d_c) \tag{4.24}$$

$e = \max(e_f, e_0)$ for [e_f et e_0: **see Figure 4.2**].

where:

e_f is the maximum eccentricity of the surface pile during filling;

e_o is the eccentricity of the center of the outlet;

C_{op} is the patch load solid reference factor for the solid.

The resulting characteristic value of the vertical force (compressive) in the wall nzSk per unit length of perimeter during discharge at any depth *z* should be determined as:

$$n_{zSK} = \int_0^z p_{we}\, dz = C_w\, \mu\, p_{h0}\left[\, z - z_0\, Y_J(z)\right] \tag{4.25}$$

b. Discharge patch load

The discharge patch load shall be used to represent accidental asymmetries of loading during discharge, as well as inlet and outlet eccentricities.

For silos in Action Assessment Class 1, the discharge patch load may be ignored.

For silos in Action Assessment Classes 2 and 3, the method of this section should be used to assess discharge loads.

The reference magnitude of the discharge outward patch pressure P_{pe} should be determined as:

$$p_{pe} = C_{pe}\, p_{he} \tag{4.26}$$

- For $h_c/d_c > 1.2$, C_{pe} equals to:

$$C_{pe} = 0.42C_{op}[1 + 2E^2](1 - e^{\{-1.5[(h_c/d_c)-1]\}}) \quad (4.27)$$

- For $h_c/d_c \leq 1.2$, C_{pe} equals to:

$$C_{pe} = \max\{0\,,0.272C_{op}([(h_c/d_c) - 1] + E)\} \quad (4.1)$$

$$E = 2\,e/\,d_c \quad (4.28)$$

$$e = \max\,(e_f,\, e_0) \quad (4.29)$$

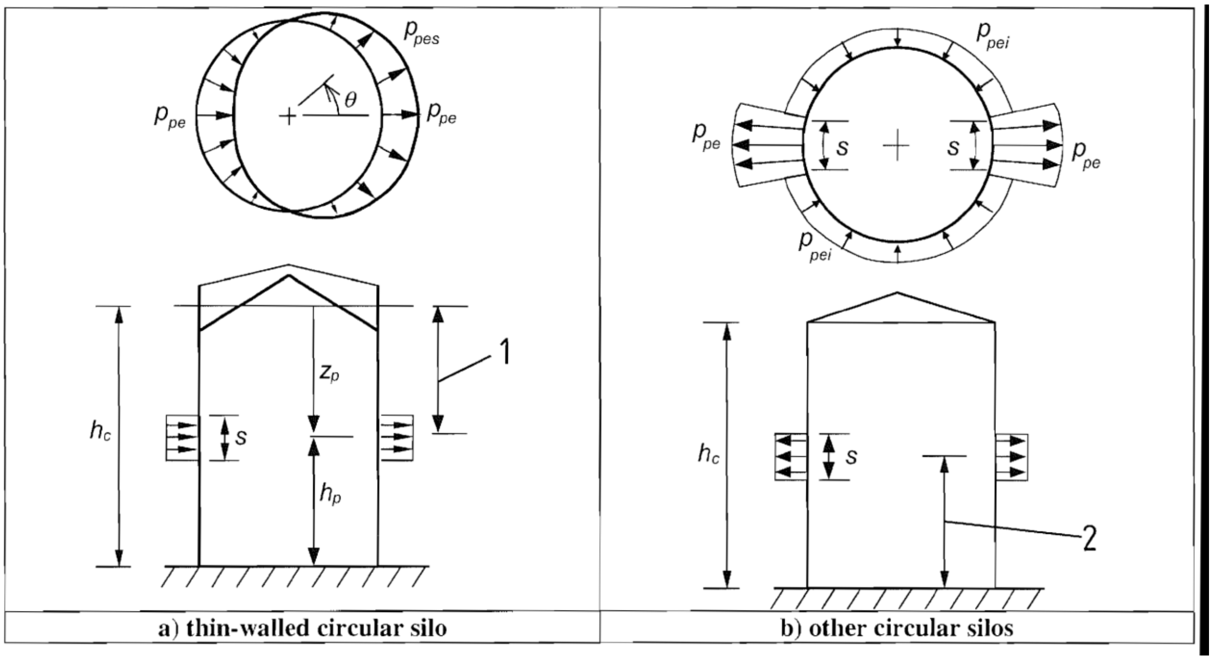


Fig. 4.6. Discharge patch loads.

➢ **Discharge patch load: thick-walled circular silos**

$$p_{pei} = p_{pe}/7 \quad (4.30)$$

➢ **Discharge patch load: thin-walled circular silos**

$$p_{pes} = p_{pe}\cos\theta \quad (4.31)$$

where:

P_{pe} is the outward patch pressure;

θ is the circumferential coordinate (see **Figure 4.6a**),

➢ **Discharge patch load: non-circular silos**

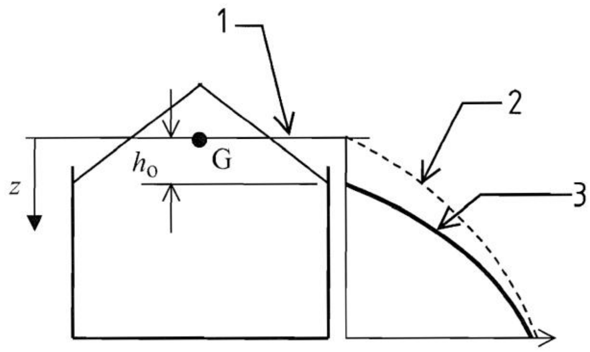
$$p_{pe,nc} = 0.36p_{pe} \quad (4.32)$$

2.2.2. Squat and intermediate slenderness silos

2.2.2.1.Filling loads

a. Symmetrical filling load

The symmetrical filling load (**see Figure 4.7**) should be calculated as follows:



Key

1. Equivalent surface
2. Slender silo rule
3. Squat silo pressures

Fig. 4.7. Filling pressures in a squat or intermediate slenderness silo

The values of horizontal pressure P_{hf} and wall frictional traction P_{wf} at any depth after filling should be determined as:

$$p_{hf} = p_{h0} Y_R \quad (4.33)$$

$$p_{wf} = \mu\, p_{hf} \quad (4.34)$$

$$p_{vf}(z) = \frac{p_{h0}}{K} Y_J(z) \quad (4.35)$$

where:

$$p_{h0} = \gamma\, K\, z_0 = \gamma\, \frac{1}{\mu}\, \frac{A}{U} \quad (4.36)$$

$$z_0 = \frac{1}{K\mu}\, \frac{A}{U} \quad (4.37)$$

$$Y_R = \left(1 - \left\{\left(\frac{z-h_0}{z_0-h_0}\right) + 1\right\}^n\right) \quad (4.38)$$

$$n = -(1 + tan\Phi_r)\left(1 - \frac{h_0}{z_0}\right) \quad (4.39)$$

where:

h_0 is the value of z at the highest solid-wall contact.

For a symmetrically filled circular silo of radius *r*, h_o should be determined as: $h_0 = (r/3) \tan \varphi_r$

And for a symmetrically filled rectangular silo of characteristic dimension d_c, h_o should be determined as: $h_0 = (d_c/4) \tan \varphi_r$

where φ_r is the angle of repose of the solid.

The value of vertical pressure P_{vf} at any depth after filling should be determined as:

$$p_{vf} = \gamma z_v \tag{4.40}$$

in which:

$$z_v = h_0 - \frac{1}{(n+1)}\left(z_0 - h_0 - \frac{(z+z_0-2h_0)^{n+1}}{(z_0-h_0)^2}\right) \tag{4.41}$$

The resulting characteristic value of the vertical force (compressive) in the wall n_{zSk} per unit length of perimeter at any depth *z* should be determined as:

$$n_{zSK} = \int_0^z p_{wf}(z)\, dz = \mu\, p_{h0}[z - z_v] \tag{4.42}$$

b. Filling path load

The filling patch load should be considered to act on any part of the silo wall.

For squat silos ($h_c/d_c \leq 1.0$) in all Action Assessment Classes, the filling patch load need not be considered ($C_{pf} = 0$).

For silos of intermediate slenderness ($1 < h_c/d_c < 2$) in Action Assessment Class I, the filling patch load may be ignored.

For silos of intermediate slenderness ($1{,}0 < h_c/d_c < 2{,}0$) in Action Assessment Classes 2 and 3, the filling patch pressure P_{pf} taken from 2.2.1.1.b should be used to represent accidental asymmetries of loading and small eccentricities of filling e_f.

For silos of squat or intermediate slenderness ($h_c < 2{,}0$) in Action Assessment Classes 2 and 3, where the eccentricity of filling e_f exceeds the critical value $e_{f,cr} = 0{,}25 d_c$, the additional load case for large filling eccentricities in squat silos should be used (See 5.3.3 of Eurocode 1 EN 1991-4:2006: E).

2.2.2.2.**Discharge loads**

a. Symmetrical discharge load

Symmetrical increases in the discharge load shall be used where it is necessary to represent possible transitory increases in pressure during the discharge process.

For squat silos *($h_c/d_c \leq 1.0$)*, the symmetrical discharge loads may be taken as identical to the filling loads.
For silos of intermediate slenderness ($1 < h_c/d_c < 2$), the symmetrical discharge pressures should be determined as:

$$p_{he} = C_h\, p_{hf} \tag{4.43}$$

$$p_{we} = C_w\, p_{wf} \tag{4.44}$$

where:
C_h and C_w are discharge factors calculated as follows:

For silos in all Action Assessment Classes that are unloaded from the top (no flow within the stored solid): $C_h = C_w = 1$.

For intermediate slenderness silos in Action Assessment Class 2 and 3, the discharge factors should be taken as:

$$C_h = 1.0 + 0.15\, C_s \tag{4.45}$$

$$C_w = 1.0 + 0.1\, C_s \tag{4.46}$$

$$C_s = (h_c/d_c) - 1 \tag{4.47}$$

where:
C_s is the slenderness adjustment factor.

For intermediate slenderness silos in Action Assessment Class 1, where the mean value of the material properties *K* and *μ* have been used for design, the discharge factors should be taken as:

$$C_h = 1.0 + [0.15 + 1.5\,(1 + 0.4e/d_c)\, C_{op}]\, C_s \tag{4.48}$$

$$C_w = 1 + 0.4\,(1 + 1.4e/d_c)\, C_s,\ e = \max\,(e_f, e_0) \tag{4.49}$$

The resulting characteristic value of the discharge vertical force (compressive) in the wall n_{zSk} per unit length of perimeter at any depth should be determined as:

$$n_{zSK} = \int_0^z p_{we}\, dz = C_w\, \mu\, p_{h0}[z - z_v] \tag{4.50}$$

b. Discharge patch load

The discharge patch pressure P_{pe} should be used to represent accidental asymmetries of loading. The rules set out in 2.2.1.2 should be used to define the form, location and magnitude of the patch load.

For squat or intermediate slenderness silos *(hc/dc* < 2,0) in all Action Assessment Classes, where the eccentricity of discharge e_o exceeds the critical value $e_{o,cr} = 0{,}25dc$, the additional load case defined in 5.3.4 of Eurocode 1 EN 1991-4:2006: E should also be adopted.

For squat silos *(hcldc* ≤ 1,0) in all Action Assessment Classes and with discharge eccentricity eo less than $e_{o,cr} = 0{,}1dc$ the discharge patch load should not be considered ($C_{pe} = 0$).

For squat or intermediate slenderness silos *(h/dc* < 2,0) in Aetion Assessment Class 1, the discharge patch load should not be considered ($C_{pe} = 0$).

For squat silos in Action Assessment Class 2 and with discharge eccentricity e_o greater than $e_{o,cr} = 0{,}1dc$ the provisions of 5.3.2.3 (Eurocode 1 EN 1991-4:2006: E) should be adopted.

For silos of intermediate slenderness (1,0 < hc/dc < 2,0) in Action Assessment Class 2, the provisions of 5.3.2.3 (Eurocode 1 EN 1991-4:2006: E) should be adopted.

For squat silos in Action Assessment Class 3 and with discharge eccentricity e_o greater than $e_{o,cr} = 0{,}1\ dc$, the provisions of 5.2.2.2 to 5.2.2.5, as appropriate, should be adopted.

For silos of intermediate slenderness in Action Assessment Class 3, the provisions of 5.2.2.2 to 5.2.2.5, as appropriate, should be adopted.

2.2.3. Retaining silos

2.2.3.1.Filling loads

a. Symmetrical filling load

The characteristic value of the horizontal pressure P_h on a vertical wall is given as follows:

$$p_h = \gamma K\ z_s[1 + sin\Phi_r] \tag{4.51}$$

where:

Zs is the depth below the highest stored solid contact with the wall (**see Figure 4.8**);

r is the upper characteristic value of the unit weight of the solid;
K is the upper characteristic value of the lateral pressure ratio for the solid;
Φr is the angle of repose of the stored solid.

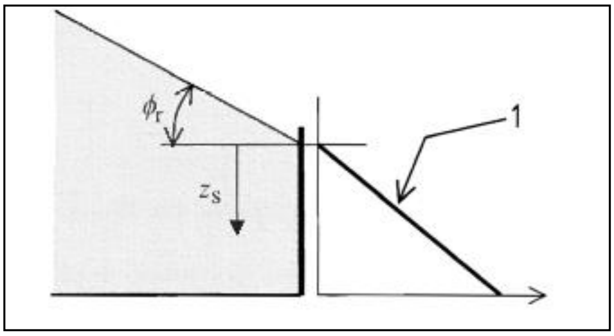


Fig. 4.8. Filling pressures in a retaining silo

The characteristic value of the resulting vertical force n_{zSk} (compressive) in the wall per unit length of circumference at any depth Z_s below the point of highest wall contact should be determined in a manner that is consistent with the pressures defined previously and the wall friction coefficient μ.

$$n_{zSK} = \gamma \frac{\mu K}{2} [1 + sin\Phi_r] {z_s}^2 \quad (4.52)$$

where μ is the upper characteristic value of the wall friction coefficient of the solid.

2.2.3.2.Discharge loads
The discharge load on the vertical wall may be taken to be less than the filling load.
With regard to 2.2.3.1 the evaluation of the conditions of discharge should take account of the possibility of unsymmetrical pressures as a result of uneven removal of solid from within the silo.

2.3.Loads on silo hoppers and silo bottoms

The characteristic values of the filling and discharge loads on silo bottoms, which are prescribed in this section for the following types of silos, shall be used:

- Flat bottoms;
- Steep hoppers;
- Shallow hoppers.

The hopper apex half angle:

$$\tan \beta < \frac{1-K}{2\,\mu_h} \tag{4.53}$$

where:

K is the lower characteristic value of the lateral pressure ratio on the vertical walls;

β is the hopper apex half angle;

μ_h is the lower characteristic value of wall I friction coefficient in the hopper.

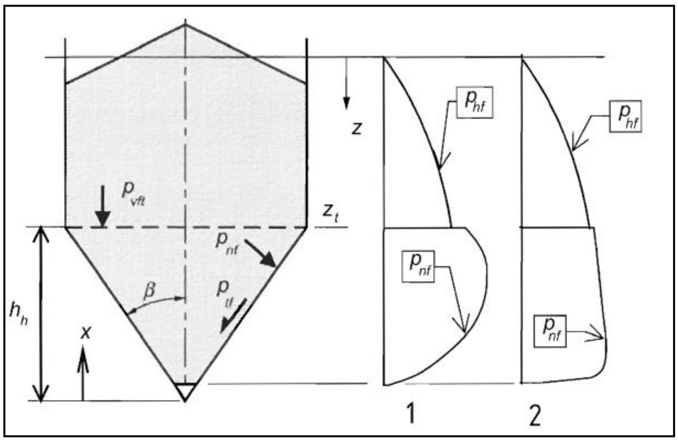


Fig. 4.9. Distributions of filling pressures in steep and shallow hoppers (1 Steep, 2 Shallow)

The mean vertical pressure at the transition between the vertical walled segment and the hopper or on the silo bottom should be determined as:

$$p_{vft} = C_b\, p_{vf} \tag{4.54}$$

where:

P_{yf} is the filling value of the vertical pressure

C_b is a bottom load magnifier to account for the possibility of larger loads being transferred to the hopper or bottom from the vertical walled segment.

For each condition in a hopper, the mean vertical stress in the solid at height x above the apex of the hopper (**see Figure 4.9**) should be determined as:

$$p_v = \left(\frac{\gamma h_h}{n-1}\right)\left\{\left(\frac{x}{h_h}\right) - \left(\frac{x}{h_h}\right)^n\right\} + p_{vft}\left(\frac{x}{h_h}\right)^n \tag{4.55}$$

The hopper pressure exponent, n, is given by:

$$n = S(F\,\mu_{heff}\cos\,\beta + F) - 2 \tag{4.58}$$

where:

- $S = 2$ for conical and square pyramidal hoppers
- $S = 1$ for wedge hoppers
- $S = 1 + \frac{b}{a}$ for rectangular-planform hoppers
- γ is the upper characteristic value of the solid's unit weight
- h_h is vertical height between hopper apex and transition (**see Figure 4.9**)
- x is the vertical coordinate upward from hopper apex (**see Figure 4.9**)
- μ_{heff} is the effective or mobilized characteristic wall friction coefficient for the hopper
- S is the hopper shape coefficient
- F is the characteristic value of the hopper pressure ratio
- β is the hopper apex half-angle (= $90° - \alpha$), or the steepest slope for a square or rectangular pyramid
- p_{vft} is the mean vertical stress in the solid at the transition after filling
- a is the length of the rectangular planform
- b is the width of the rectangular planform

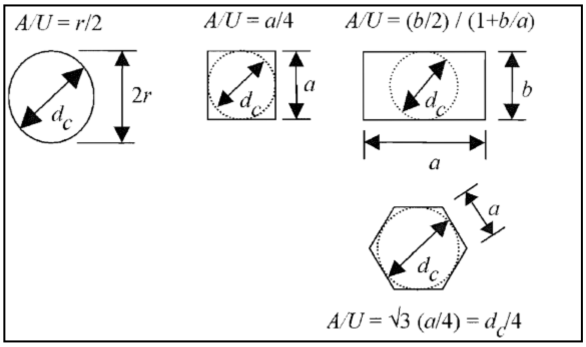


Fig. 4.10

2.3.1. Flat bottoms

2.3.1.1. Vertical Pressures on Flat Bottoms in Slender Silos

The vertical pressure acting on a flat bottom (inclination α ≤ 5°) may be taken as uniform, except when the silo is squat or of intermediate slenderness. The vertical pressure p_v acting on a flat bottom shall be determined as:

$$p_v = p_{vft} \tag{4.59}$$

The vertical pressure acting on a flat bottom during discharge shall be taken as identical to the vertical pressure at the end of filling.

2.3.1.2.Vertical Pressures on Flat Bottoms in Squat and Intermediate Silos

The vertical pressure p_{vsq} acting on the flat bottom may be taken as:

$$p_{vsq} = p_{vb} + \Delta p_{sq}(\frac{2.0 - h_c/d_c}{2.0 - h_{tp}/d_c}) \tag{4.60}$$

in which:

$$\Delta p_{sq} = p_{vtp} - p_{vho} \tag{4.61}$$

$$p_{vtp} = \gamma\, h_{tp} \tag{4.62}$$

where:

- p_{vb} is the uniform component of vertical pressure.
- p_{vho} is the Janssen vertical pressure at the base of the top pile.
- h_o is the depth below the equivalent surface of the base of the top pile, defined as the lowest point on the wall not in contact with the stored solid;
- h_{tp} is the total height of the top pile, i.e., vertical distance from the lowest point of wall not in contact with the stored solid to the highest stored particle;
- $h_c$is the depth of the silo base below the equivalent surface;
- γ is the unit weight of the stored solid;
- d_c is the silo diameter.

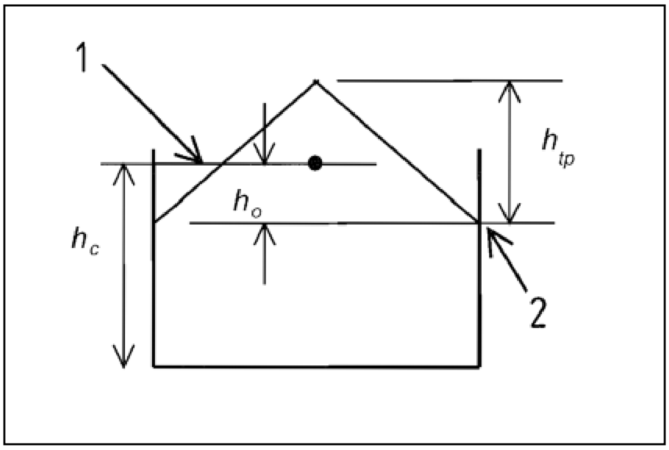


Fig. 4.11

2.3.2. Steep hoppers

2.3.2.1.Mobilized Friction

For both filling and discharge conditions, the effective or mobilized wall friction coefficient shall be taken as:

$$\mu_{\text{heff}} = \mu_h \tag{6.63}$$

where:

$\mu_h$is the lower characteristic value of the wall friction coefficient in the hopper.

2.3.2.2.Filling Loads

Under filling conditions, the mean vertical stress $p_v$in the stored solid at any level in a steep hopper shall be determined, with the parameter $F = F_f$, where:

$$F_f = 1 - \frac{b}{1 + \frac{\tan\beta}{\mu_h}} \tag{4.64}$$

The parameter n is then given by:

$$n = S(1 - b)\,\mu_h \cot\,\beta \tag{4.65}$$

where:

- b= empirical coefficient, $b = 0.2$
- S= hopper shape coefficient

- β= hopper half-angle

The normal pressure $p_{nf}$and frictional traction p_{tf}at any point on the wall of a steep hopper after filling (**see Figure 4.9**) should be determined as:

$$p_{nf} = F_f \, p_v \tag{4.66}$$

$$p_{tf} = \mu_h \, F_f \, p_v \tag{4.67}$$

2.3.2.3.Discharge Loads

Under discharge conditions, the mean vertical stress $p_v$in the stored solid at any level in a steep hopper shall again be determined, with $F = F_e$.

The value of $F_e$may be calculated using the following expression:

$$F_e = \frac{1 + \sin\ \phi_i \cos\ \varepsilon}{1 - \sin\ \phi_i \cos\ (2\beta + \varepsilon)} \tag{4.68}$$

in which:

$$\varepsilon = \phi_{wh} + \sin^{-1}(\frac{\sin \phi_{wh}}{\sin \phi_i}) \tag{4.69}$$

$$\phi_{wh} = \tan^{-1} \mu_h \tag{4.70}$$

where:

- μ_h= lower characteristic value of wall friction coefficient in the hopper;
- ϕ_i= angle of internal friction of the stored solid.

The normal pressure $p_{ne}$and frictional traction p_{te}(**see Figure 4.12**) at any point on the wall of a steep hopper during discharge are determined as:

$$p_{ne} = F_e \, p_v \tag{4.71}$$

$$p_{te} = \mu_h \, F_e \, p_v \tag{4.72}$$

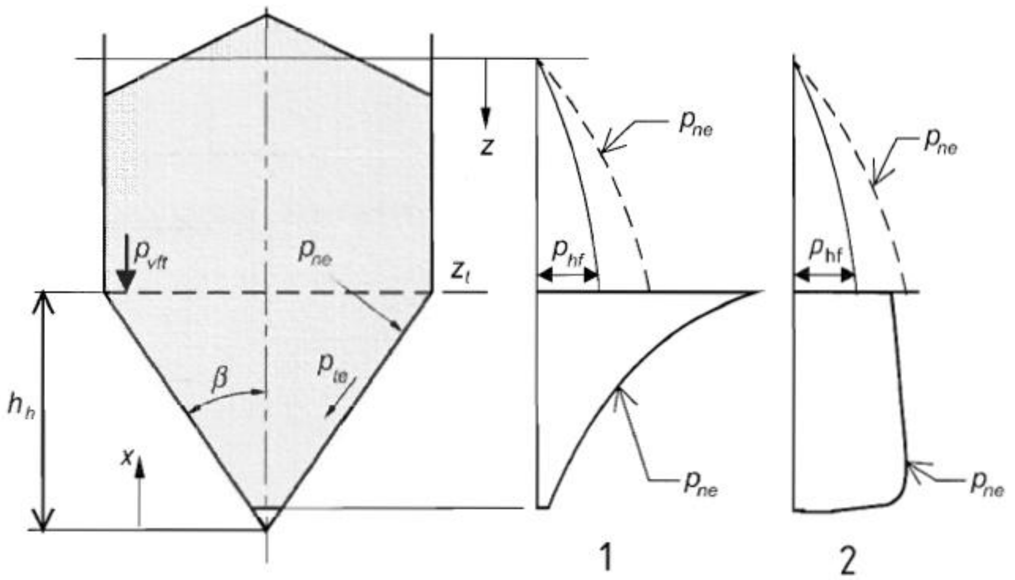


Fig. 4.12. Discharge pressures in steep and shallow hoppers (1 Steep, 2 Shallow)

2.3.3. Shallow hoppers

2.3.3.1.Mobilized Friction

In a shallow hopper, the wall friction is not fully mobilized. The mobilized or effective wall friction coefficient shall be determined as:

$$\mu_{\text{heff}} = \frac{(1-K)}{2\tan\ \beta} \tag{4.73}$$

where:

- K= lower characteristic value of the lateral pressure ratio for the vertical section;
- β= half-angle of the hopper.

2.3.3.2.Filling Loads

Under filling conditions, the mean vertical stress $p_v$in the stored solid at any level of a shallow hopper shall be determined with the value of the parameter Fgiven by:

$$F_f = 1 - \{\frac{b}{1+\tan\ \beta/\mu_{\text{heff}}}\} \tag{4.74}$$

The parameter n is then given by:

$$n = S(1-b)\ \mu_{\text{heff}}\cot\ \beta \tag{4.75}$$

where:

- μ_{heff}= mobilized or effective wall friction coefficient in the shallow hopper,
- b= empirical coefficient, $b = 0.2$,

The normal pressure $p_{nf}$and frictional traction p_{tf}at any point on the wall of a shallow hopper after filling shall be determined as:

$$p_{nf} = F_f \, p_v \tag{4.76}$$

$$p_{tf} = \mu_{\text{heff}} F_f \, p_v \tag{4.77}$$

2.3.3.3.Discharge Loads

In shallow hoppers under discharge conditions, the normal pressure and frictional traction may be taken as identical to the values during filling

3. APPLICATION “Circular RC silo with a steep conical hopper”

3.1.Geometry & materials

- Internal diameter $D = 5.0$ m ⇒ $r = 2.50$ m
- Cylindrical wall thickness $t = 250$ mm
- Cylindrical wall height (above transition) $H_c = 12$ m
- Hopper height (apex to transition) $h_h = 3$ m
- Hopper half-angle $\beta = 45°$(steep hopper → §6.3)
- Concrete C30/37, steel $f_{yk} = 400$ MPa ⇒ $f_{yd} = 400/1.15 = 348$ MPa
- Bulk unit weight $\gamma = 25$ kN/m^3
- Friction parameters (typical powders): $\varphi_i \approx 35° \Rightarrow K = 1 - \sin\ \varphi_i \approx 0.426$

  Wall friction angle $\varphi_{wh} \approx 20° \Rightarrow \mu_h = \tan\ \varphi_{wh} \approx 0.364$
- Action factor: $\gamma_G = 1.35$

3.2.Cylinder pressures (reference check at base of cylinder)

Using **Janssen** with $\lambda = z_0 = r/(2K\mu) = 2.50/(2 \cdot 0.426 \cdot 0.364) = 8.06$ m.

Vertical stress at **transition level** (top of hopper), depth $z = H_c = 12$ m:

$$p_v(H_c) = \gamma\,\lambda(1 - e^{-H_c/\lambda}) = 25 \times 8.06\,(1 - e^{-12/8.06}) \approx 156 \text{ kPa.}$$

Normal wall pressure in **cylinder** just above transition:

$$p_{n,cyl} = K\,p_v \approx 0.426 \times 156 = 66.4 \text{ kPa.}$$

Hoop membrane force (per metre height):

$$T_{k,cyl} = p_{n,cyl}\,r = 66.4 \times 2.5 = 166 \text{ kN/m}, T_{d,cyl} = 1.35\,T_k \approx 224 \text{ kN/m.}$$

Required hoop steel there:

$$A_{s,\text{req,cyl}} = \frac{T_{d,cyl}}{f_{yd}} = \frac{224\,000}{348 \times 10^6} = 644\text{mm}^2/\text{m} = 6.44\text{cm}^2$$

(This reproduces the cylinder-only logic; keep the earlier $\emptyset 12@150$*away from the transition.)*

3.3.Hopper effects at the transition (EN 1991-4 §6.3 Steep hoppers)

For **filling** (§6.3.2):

$$F_f = 1 - \frac{b}{1 + \frac{\tan\beta}{\mu_h}}, b = 0.2, \beta = 45°, \mu_h = 0.364 \Rightarrow. \qquad (6.17)$$

$$F_f = 1 - \frac{0.2}{1 + 1/0.364} = 0.947$$

Normal and tangential wall actions at the **hopper wall** (after filling) (§6.3.2):

$$p_{nf} = F_f\,p_v, p_{tf} = \mu_h\,F_f\,p_v. \qquad (6.19–6.20)$$

At the transition we use the same **mean vertical stress** $p_v$as above (end of filling): $p_v = 156$ kPa.

$$p_{nf} = 0.947 \times 156 = 148 \text{ kPa}, p_{tf} = 0.364 \times 0.947 \times 156 = 53.5 \text{ kPa.}$$

Horizontal line load from the hopper on the transition ring

Resolve the hopper wall actions to a **horizontal** component on the circular ring:

$$q_h = p_{nf} \sin \beta + p_{tf} \cos \beta (\beta = 45°)$$

$$q_h = 148(0.707) + 53.5(0.707) \approx 142 \text{ kPa.}$$

The **equivalent hoop force** per metre height in the ring (radius r):

$$T_{k,hop} = q_h \, r = 142 \times 2.50 = 356 \text{ kN/m}, T_{d,hop} = 1.35 \, T_{k,hop} \approx 480 \text{ kN/m.}$$

Required **hoop steel due to hopper**:

$$A_{s,\text{req,hop}} = \frac{T_{d,hop}}{f_{yd}} = \frac{480\,000}{348 \times 10^6} = \boxed{1\,380 \text{mm}^2/\text{m}} = 13.80 \text{cm}^2$$

For **discharge** use §6.3.3: compute $F_e$from (6.21)–(6.23). Then

$p_{ne} = F_e p_v$, $p_{te} = \mu_h F_e p_v$; repeat the resolution $q_h = p_{ne} \sin \beta + p_{te} \cos \beta$. With typical powders $F_e$is close to F_f, so the **envelope** is often the filling case above.

3.4.Design envelope at/just above the transition

Take the governing of:

- **Cylinder hoop** $T_{d,cyl} \approx 224$ kN/mand
- **Hopper-induced ring** $T_{d,hop} \approx 480$ kN/m.

The **transition** is governed by hopper effects:

$$A_{s,\text{req}} = \boxed{1\,380 \text{ mm}^2/\text{m}} \text{(provide within 1–2 m above the junction).}$$

A practical reinforcement choice

- **Hoop (circumferential) rings near transition:**

  $\emptyset 12@80$ mm $\Rightarrow A = \frac{113}{0.08} = 1\,410 \text{ mm}^2/\text{m} \geq 1\,380.$

  (Alternatively, $\emptyset 14@100$ mm $\Rightarrow 1540 \text{ mm}^2/\text{m}$*).*

- **Away from transition (upper wall):** keep $\emptyset 12@150$ mm (≈ 754 mm²/m), which exceeds the cylinder-only need (≈ 644 mm²/m).

- **Vertical steel (crack/temperature):** Ø10@150 mmeach face (≈ 523 mm²/m per face), or verify by EC2 crack-width limits.

3.5.Notes on scope and refinements

- This inserts **hopper effects per EN 1991-4** in a clean way by resolving p_{nf}, p_{tf}into a **horizontal ring load** at the transition.
- If your hopper is **shallow**, use $\mu_{\text{heff}}$from **(6.26)** and $F_f$from **(6.27)**; for discharge, usually **6.4.3** allows using the same values as filling.
- For a detailed shell design, you may also check **meridional compression** in the hopper wall and the **ring girder** at the junction.
- Always combine with **global loads** (wind/seismic), local patch loads, and check base slab/foundation separately.

Final reinforcement (including hopper effects)

- **Transition zone (≈ first 1–2 m above junction):** Ø12@80 mm circumferential rings
- **Upper cylinder:** Ø12@150 mm
- **Vertical steel (both faces):** Ø10@150 mm

CHAPTER 05: RC BRIDGES

1. GENERALITIES

1.1.Definition

The basic purpose of a bridge is to carry traffic over an opening or discontinuity in the landscape. Various types of bridge traffic can include pedestrians, vehicles, pipelines, cables, water, and trains, or a combination thereof.

An opening can occur over a highway, a river, a valley, or any other type of physical obstacle. The need to carry traffic over such an opening defines the function of a bridge. The design of a bridge can only commence after its function has been properly defined.

Therefore, the process of building a bridge is not initiated by the bridge engineer. Just like roads or a drainage system, or other types of infrastructure, a bridge is a part of a transportation system and a transportation system is a component of a city's planning efforts or its area development plan. The function of a bridge must be defined in these master plans.

The raw materials of concrete, consisting of water, fine aggregate, coarse aggregate, and cement, can be found in most areas of the world and can be mixed to form a variety of structural shapes. The great availability and flexibility of concrete material and reinforcing bars have made the reinforced concrete bridge a very competitive alternative.

Reinforced concrete bridges may consist of precast concrete elements, which are fabricated at a production plant and then transported for erection at the job site, or cast-in-place concrete, which is formed and cast directly in its setting location.

Cast-in-place concrete structures are often constructed monolithically and continuously. They usually provide a relatively low maintenance cost and better earthquake-resistance performance. Cast-in-place concrete structures, however, may not be a good choice when the project is on a fast-track construction schedule or when the available falsework opening clearance is limited.

1.2.RC Bridge Types

Reinforced concrete sections, used in the bridge superstructures, usually consist of slabs, T-beams (deck girders), and box girders (**Figure 5.1**). Safety, cost-effectiveness, and aesthetics are generally the controlling factors in selecting the proper type of bridge.

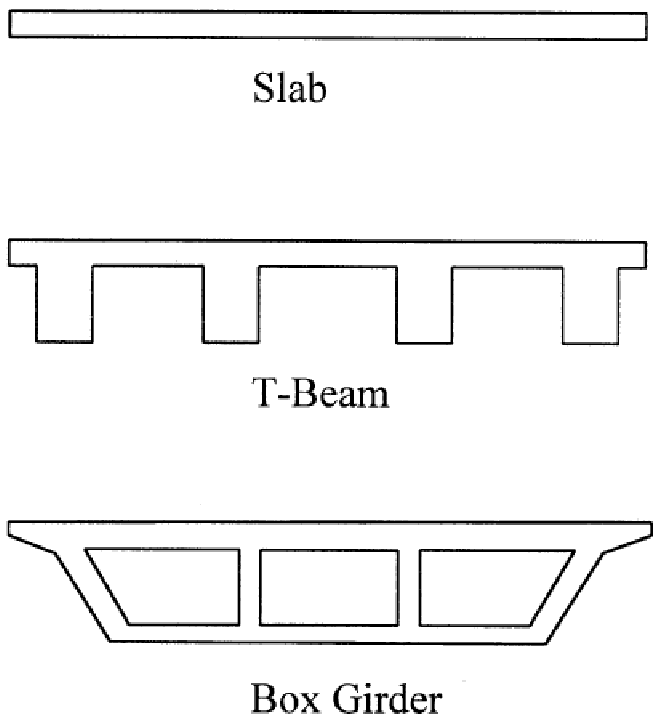


Fig. 5.1. Bridge types.

1.2.1. Slab Bridges

Longitudinally reinforced slab bridges have the simplest superstructure configuration and the neatest appearance. They generally require more reinforcing steel and structural concrete than do girder-type bridges of the same span. However, the design details and formworks are easier and less expensive. It has been found economical for simply supported spans up to 9 m and for continuous spans up to 12 m.

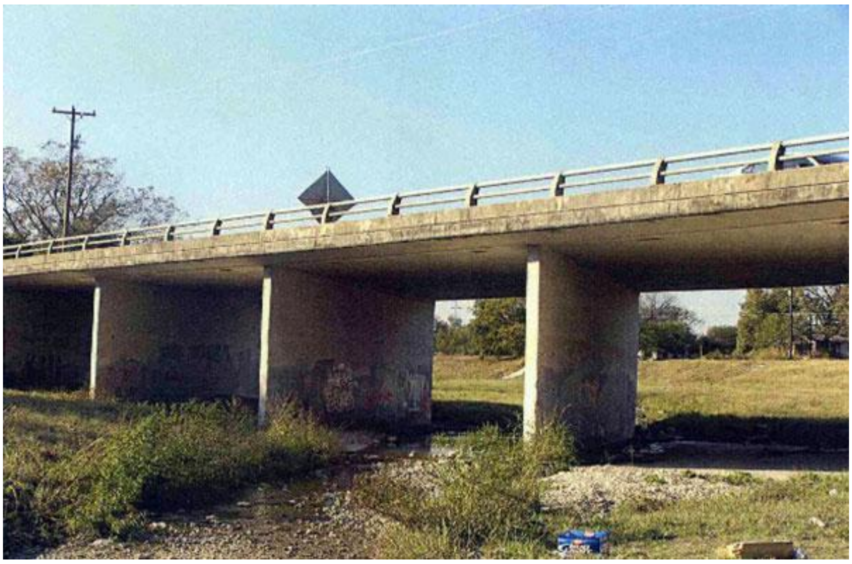


Fig. 5.2. Slab Bridge.

1.2.2. T-Beam Bridges

The T-beam construction consists of a transversely reinforced slab deck which spans across to the longitudinal support girders. These require a more-complicated formwork, particularly for skewed bridges, compared to the other superstructure forms. T-beam bridges are generally more economical for spans of 12 to 18 m. The girder stem thickness usually varies from 35 to 55 cm and is controlled by the required horizontal spacing of the positive moment reinforcement. Optimum lateral spacing of longitudinal girders is typically between 1.8 and 3.0 m for a minimum cost of formwork and structural materials. However, where vertical supports for the formwork are difficult and expensive, girder spacing can be increased accordingly.

Fig. 5.3. T beam Bridge.

1.2.3. Box-Girder Bridges

Box-girder bridges contain top deck, vertical web, and bottom slab and are often used for spans of 15 to 36 m with girders spaced at 1.5 times the structure depth. Beyond this range, it is probably more economical to consider a different type of bridge, such as post-tensioned box girder or steel girder superstructure. This is because of the massive increase in volume and materials. They can be viewed as T-beam structures for both positive and negative moments. The high torsional strength of the box girder makes it particularly suitable for sharp curve alignment, skewed piers and abutments, superelevation, and transitions such as interchange ramp structures.

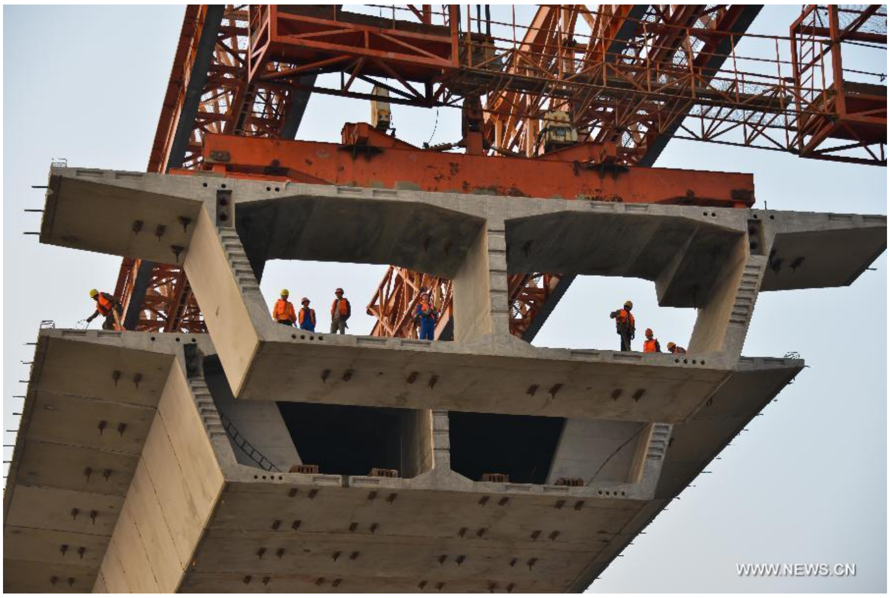


Fig. 5.4. Box Bridge.

1.3.Main Components

1. **Superstructure** — deck, slab or girders, parapets
2. **Substructure** — piers, abutments, bearings
3. **Foundation** — spread footings, piles, or caissons

2. DESIGN CONSIDERATIONS

2.1.Basic Design Theory

Limit state is defined as the limiting condition of acceptable performance for which the bridge or component was designed. In order to achieve the objective for a safe design, each bridge member and connection is required to examine some, or all, of the service, fatigue, strength, and extreme event limit states. All applicable limit states shall be considered of equal importance.

2.2.Design Limit States

2.2.1. Service Limit States

For concrete structures, service limit states correspond to the restrictions on cracking width and deformations under service conditions. They are intended to ensure that the bridge will behave and perform acceptably during its service life. The tensile stress in the steel reinforcement at the service limit state should not exceed:

$$f_{sa} = \frac{Z}{(d_c A)^{1/3}} \leq 0.6 f_y \tag{5.1}$$

where

f_{sa}	Allowable service stress in the reinforcement	MPa
Z	Section modulus or load effect coefficient (depends on geometry and loading)	MPa
d_c	Effective cover to the steel centroid (distance from the compression face to centroid of reinforcement)	mm
A	Effective tension area of concrete surrounding one bar (≈ spacing × cover)	mm²
f_y	Yield strength of reinforcement steel	MPa

For superstructures with constant depth, **Table 5.1** shows the typical minimum depth recommendation for a given span length.

Table 5.1. Traditional Minimum Depths for Constant Depth Superstructures.

Bridge Type	**Minimum Depth (Including Deck)**	
	Simple Spans	**Continuous Spans**
Slab bridges	$h_{min} = \frac{1.2\,(S + 3000)}{30}$	$h_{min} = \frac{S + 3000}{30} \geq 165$ mm
T-beam bridges	$h_{min} = 0.070\,L$	$h_{min} = 0.065\,L$
Box-girder bridges	$h_{min} = 0.060\,L$	$h_{min} = 0.055\,L$
Pedestrian bridges	$h_{min} = 0.035\,L$	$h_{min} = 0.033\,L$

S (mm) is the slab span length and *L* (mm) is the span length.

2.2.2. Fatigue Limit States

Fatigue limit states are used to limit stress in steel reinforcements to control concrete crack growth under repetitive truck loading in order to prevent early fracture failure before the design service life of a bridge. Fatigue loading consists of one design truck with a constant spacing of 9000 mm between the 145-kN axles. Fatigue is considered at regions where compressive stress due to permanent loads is less than two times the maximum tensile live-load stress resulting from the fatigue-load combination.

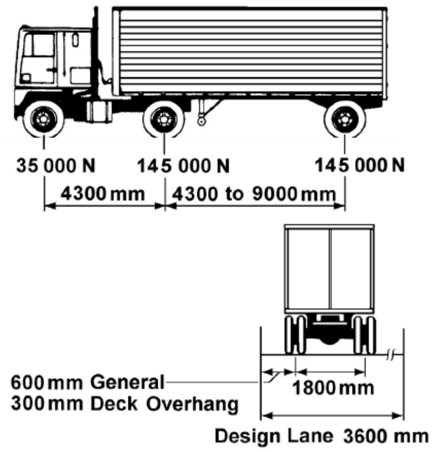


Fig. 5.5. Characteristics of the Design Truck

2.2.3. Strength Limit States and Extreme Event Limit States

For reinforced concrete structures, strength and extreme event limit states are used to ensure that strength and stability are provided to resist specified statistically significant load combinations.

2.3.Flexural Strength

For T beams (**Fig. 5.6**), the nominal flexural strength is:

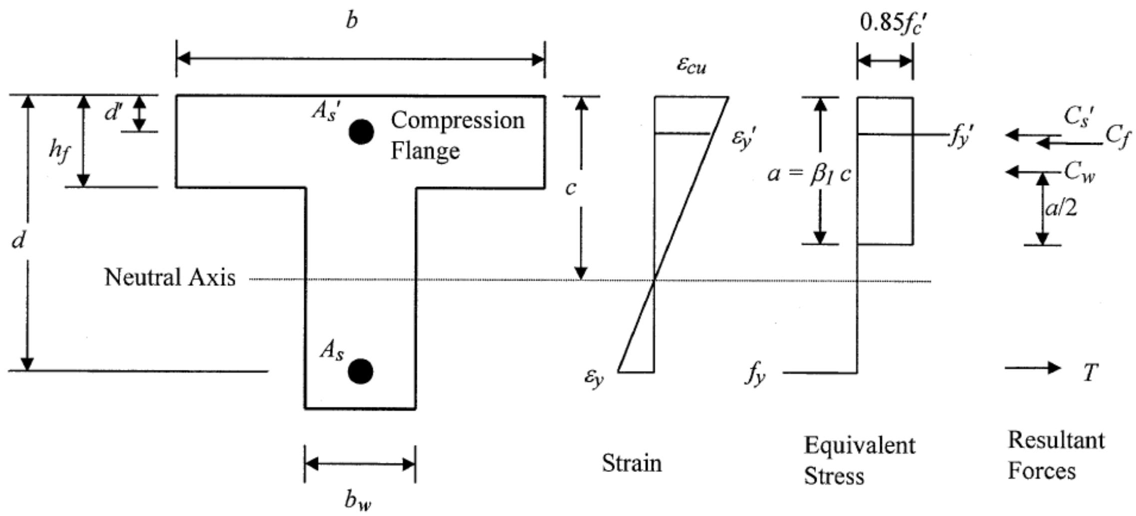


Fig. 5.6. Reinforced concrete beam when flexural strength is reached.

Flexural strength of a T-beam (positive bending)

$$\boxed{M_n = A_s f_y\,(d-\tfrac{a}{2}) + A_s' f_y'\,(\tfrac{a}{2}-d') + 0.85\,\beta_1 f_c'\,(b-b_w)\,h_f\,(\tfrac{a}{2}-\tfrac{h_f}{2})} \quad (5.2)$$

where

- A_s: area of **tension** reinforcement (bottom steel)
- A_s': area of **compression** reinforcement (optional, top steel)
- f_y, f_y': design/yield strengths of the steel used for $A_s$and A_s'
- d: effective depth to A_s(to centroid of tension steel)
- d': cover to compression steel (to centroid of A_s')
- f_c': concrete cylinder/ cube strength (use the correct design value in your code)
- β_1: stress-block factor (ACI: function of f_c'; EC2 uses rectangular block with λ& η)
- b: flange (effective) width
- b_w: web (rib) width
- h_f: flange thickness (deck slab thickness participating in compression)
- $a = \beta_1 c$: equivalent rectangular-block depth, with $c$the neutral-axis depth

The first two terms are the classic "rectangular-section" couple ($A_s f_y$vs. concrete in the **web**) taken with lever arm $(d - a/2)$plus the **compression steel** couple ($A_s' f_y'$vs. same concrete web resultant) with lever arm $(a/2 - d')$.

The final term adds the **extra compression from the flange outside the web**, whose force is

$$C_f = 0.85\,\beta_1 f_c'\,(b-b_w)\,h_f \quad (5.3)$$

acting at $h_f/2$from the top; its couple with the web resultant at $a/2$produces the added moment $C_f\,(a/2 - h_f/2)$.

When to use this vs. the rectangular formula

1. Compute $a$from force equilibrium:

$$\underbrace{A_s f_y}_{T} = \underbrace{0.85\,\beta_1 f_c'\,[\,b_w(a-h_f)_+ + b\,\min(a,h_f)\,]}_{C_c} + \underbrace{A_s' f_y'}_{C_s'} \quad (5.4)$$

where $(x)_+ = \max(x,0)$.

- If $a \le h_f$: the entire compression block is within the flange → treat as rectangular with width b:

$$a = \frac{A_s f_y - A_s' f_y'}{0.85\,\beta_1 f_c'\,b} \quad (5.5)$$

$$M_n = A_s f_y\,(d-\tfrac{a}{2}) + A_s' f_y'\,(\tfrac{a}{2}-d') \quad (5.6)$$

- If $a > h_f$: **use the T-beam equation** shown above.

Effective flange width b**:** for bridges, take b_{eff}per your code (EC2/EN 1992-2 or AASHTO), typically limited by span and slab overhangs. Use $b_w$for the rib width.

2.4.Shear Strength

The **design shear stress**:

$$v_{Ed} = \frac{V_{Ed}}{b_w z} \tag{5.7}$$

Must satisfy $v_{Ed} \leq v_{Rd,c} + v_{Rd,s}$ (concrete + shear reinforcement). Shear Strength of Reinforced-Concrete Bridge Members

2.4.1. Ultimate Limit State Condition

A concrete section must satisfy

$$V_{Ed} \leq V_{Rd,c} + V_{Rd,s} \tag{5.8}$$

where

- V_{Ed}= design shear force (from load combinations),
- $V_{Rd,c}$= shear resistance without shear reinforcement (concrete contribution),
- $V_{Rd,s}$= shear resistance provided by stirrups (shear reinforcement).

2.4.2. Concrete Shear Capacity

For members with shear reinforcement (EC2 §6.2.2):

$$V_{Rd,c} = [\, C_{Rd,c}\, k\, (100\rho_l f_{ck})^{1/3} + k_1 \sigma_{cp}\,]\, b_w\, d \tag{5.9}$$

with

- $C_{Rd,c} = 0.18/\gamma_c$ (≈ 0.12 for γ_c = 1.5),
- $k = 1 + \sqrt{200/d} \leq 2.0$ (d in mm),
- $\rho_l = A_{sl}/(b_w d)$ (Longitudinal tensile reinforcement ratio, ≤ 0.02),
- σ_{cp}= mean axial stress (= N / Ac ≤ 0 for tension),
- b_w= web width, d= effective depth.

For bridge decks, $V_{Rd,c}$may be increased 10–15 % because of compressive membrane action or transverse prestress, per EN 1992-2 §6.2.3.

2.4.3. Design Shear Resistance with Stirrups

If $V_{Ed} > V_{Rd,c}$:

$$V_{Rd,s} = \frac{A_{sw}}{s}\, z\, f_{ywd} \cot\ \theta \qquad (5.10)$$

where

- A_{sw}= area of shear links (two legs if closed stirrup),
- s= spacing of stirrups,
- $z \approx 0.9d$= lever arm,
- $f_{ywd} = f_{yk}/\gamma_s$= design yield strength of stirrups,
- θ= inclination of concrete struts, $21° \leq \theta \leq 45°$.

Eurocode often assumes $\theta = 45°$ for conservative bridge design.

2.4.4. Maximum Shear Resistance

$$V_{Rd,max} = \alpha_{cw}\, b_w\, z\, v_1\, f_{cd}\, (\cot\ \theta + \tan\ \theta)^{-1} \qquad (5.11)$$

with

- $\alpha_{cw} = 1.0$(for non-prestressed),
- $v_1 = 0.6(1 - f_{ck}/250)$≤ 0.6,
- $f_{cd} = f_{ck}/\gamma_c$.

Table 5.2. Comparaison Table.

Criterion	Eurocode 2	ACI 318 / AASHTO LRFD	Comment
Concrete shear (V_c)	Empirical ∝ $(\rho f_c)^{1/3}$	$V_c = 0.17\sqrt{f_c'} b_w d$	EC2 gives variable k factor
Shear reinforcement (V_s)	$\frac{A_{sw}}{s} z f_{ywd} \cot\ \theta$	$A_v f_y d/s$	similar format
Inclined-strut angle	(21°–45°)	45° (default)	influences lever arm
Safety factors	γ_c, γ_s	ϕ = 0.75	design philosophy

2.5.Skewed Concrete Bridges

2.5.1. Definition and Geometry

A skewed bridge has its abutments or piers inclined at an angle θ (called the *skew angle*) to a line perpendicular to the traffic direction.

Typical ranges:

- θ = 0° → right bridge
- θ = 15–30° → moderately skewed
- θ ≥ 45° → highly skewed bridge

Skewness causes **non-uniform distribution of reactions and stresses**, especially near the acute corners.

Table 5.3. Structural behavior

Effect	Description	Practical Consequence
Load path distortion	Diagonal transfer of load toward the obtuse corner	Unequal bearing pressures
Torsion coupling	Skew introduces torsional moments in the slab or girder system	Additional twisting of girders
Unequal shear	Acute-corner girders carry more shear	Increase shear reinforcement in those webs
Differential deflection	Deck corners deflect differently	Cracking may concentrate at acute corners
Skew-bending interaction	Combined bending and torsion at supports	Requires 3D or grillage modeling

2.5.2. Analysis Methods

For teaching purposes, mention three common approaches:

1. **Simplified methods:** (for θ ≤ 20°)

Treat bridge as *equivalent right bridge*; increase support shear by 5 – 10 %. Use same flexural design, but apply correction factors:

$$M_{Ed,skew} = M_{Ed,rectangular}(1 + 0.1\sin^2\theta) \tag{5.12}$$

2. **Grillage analogy or FEM:** Discretize slab or deck into beam elements oriented in both directions. Captures torsion and corner effects — recommended for θ ≥ 25°.
3. **Finite-element shell models**: Used for modern Eurocode bridge design (EN 1992-2 + EN 1991-2 + EN 1994). Provides accurate stress fields, especially for box or slab bridges.

2.5.3. Design Recommendations

◇ Flexural Design

- Use design moments from the acute-corner girder (most critical).
- For moderate skew (≤ 30°), increase the design moment by ≈ 10 %.
- Reinforce the deck diaphragm to resist torsion.

◇ Shear Design

Skew increases shear near supports. Eurocode 2 does not provide explicit skew factors, but good practice is:

$$V_{Ed,skew} = V_{Ed,rectangular}\,(1 + 0.15\sin\,\theta) \qquad (5.13)$$

- Add extra stirrups in the acute-corner girders or webs.
- Limit stirrup spacing to ≤ 0.75 × the spacing of the rectangular bridge design.

◇ Bearings and Supports

- Bearings at skewed abutments must allow rotation about the bridge axis.
- Use elastomeric bearings capable of differential shear deformation.

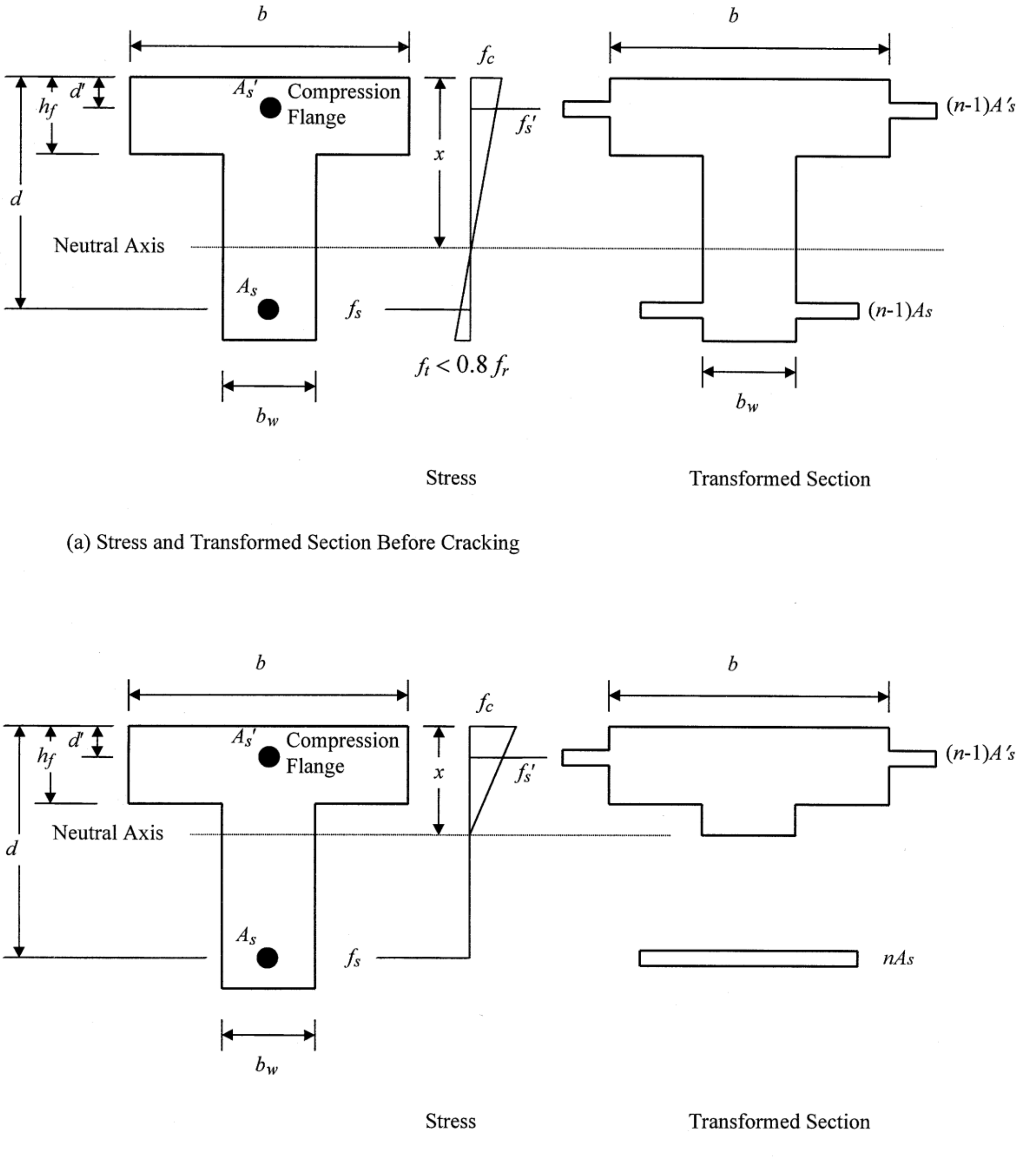
◇ Deck Reinforcement Detailing

- Provide diagonal top reinforcement across corners.
- Continue longitudinal steel into diaphragms to control cracking.
- In slab bridges, minimum reinforcement $\rho_l \geq 0.0035$ for skew ≥ 45°.

2.6.Design Information

2.6.1. Stress Analysis at Service Limit States

A reinforced concrete beam subject to flexural bending moment is shown in **Figure 5.7** and *x* is the distance between the neutral axis and the extreme compressed concrete fiber. Assume the neutral axis falls within the web ($x > hf$) and the stress in extreme tension concrete fiber is greater than 80% of the concrete modulus of rupture ($ft \geq 0.8\,fr$).



(a) Stress and Transformed Section Before Cracking

(b) Stress and Transformed Section After Cracking

Fig. 5.7. Reinforced concrete beam for working stress analysis.

2.6.2. Effective Flange Width

When reinforced concrete slab and girders are constructed monolithically, the effective flange width (b_{eff}) of a concrete slab, which will interact with girders in composite action, may be calculated as:

(a) Interior Beams

For **interior T-beams**, the effective flange width $b^I_{\text{eff}}$is determined as the *smallest* of the following three values:

$$b_{\text{eff}}^{I} = \text{the smallest of} \begin{cases} \frac{l_{\text{eff}}}{4} \\ 12t_s + b_w \\ \text{the average spacing of adjacent beams} \end{cases} \quad (5.14)$$

where:

- l_{eff}= effective span of the beam,
- t_s= thickness of the slab,
- b_w= width of the web (stem) of the beam.

This formula ensures that the effective width does not exceed realistic limits dictated by geometry and stress distribution.

(b) Exterior Beams

For **exterior T-beams**, the effective flange width $b_{\text{eff}}^{E}$is smaller and asymmetric because the beam supports a slab on only one side, while the other side extends as an overhang.
It is calculated as:

$$b_{\text{eff}}^{E} = \frac{1}{2} b_{\text{eff}}^{I} + \text{the smallest of} \begin{cases} \frac{l_{\text{eff}}}{8} \\ 6t_s + \frac{b_w}{2} \\ \text{the width of overhang} \end{cases} \quad (5.15)$$

where:

- b_{eff}^{I}= effective flange width of the interior beam,
- b_{eff}^{E}= effective flange width of the exterior beam,
- b_{oh}= width of the overhang (measured from the centerline of the exterior web to the deck edge).

where the effective span length (*l*eff) may be calculated as the actual span for simply supported spans. Also, the distance between the points of permanent load inflection for continuous spans of either positive or negative moments (*ts*) is the average thickness of the slab, and *bw* is the greater of web thickness or one half the width of the top flange of the girder.

(c) Design Significance

- The interior beams share the slab compression region with adjacent beams on both sides.
- The exterior beams have less effective width due to the free edge of the deck, which limits stress transfer.

- Using the smallest value in each case ensures safety by preventing overestimation of the beam's stiffness and bending capacity.

2.6.3. Concrete Cover

Concrete cover for unprotected main reinforcing steel should not be less than that specified in **Table 5.4** and modified for the water/cement ratio.

Table 5.4. Cover for Unprotected Main Reinforcing Steel (mm)

Situation	Minimum Cover (mm)
Direct exposure to salt water	**100**
Cast against earth	**75**
Coastal environment	**75**
Exposure to deicing salt	**60**
Deck surface subject to tire stud or chain wear	**60**
Exterior surfaces (other than above)	**50**
Interior surfaces (other than above): • Up to No. 36 bar • No. 43 and No. 57 bars	 **40** **50**
Bottom of cast-in-place (CIP) slab: • Up to No. 36 bar • No. 43 and No. 57 bars	 **25** **50**

2.7.Details of Reinforcement

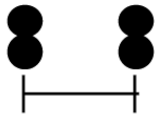
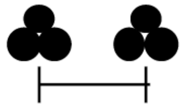
Table 5.5. shows the minimum center-to-center spacing between parallel reinforcing bars.

➢ Design Notes:

1. Minimum clear spacing shall not be less than the *maximum of*:
   - the bar diameter,
   - 25 mm,
   - or 1.50 times the maximum aggregate size.
2. When bars are placed in multiple layers or bundles, spacing should be increased to ensure concrete can flow between bars during casting.

3. For bridge decks, tighter spacing is sometimes used to control cracking due to temperature and shrinkage effects, but must still allow proper vibration of concrete.
4. Bundled reinforcement (two or more bars in contact) should follow the spacing for "in contact" arrangements shown above.
5. Adequate spacing also ensures proper durability — allowing sufficient concrete cover around each bar to resist corrosion.

Table 5.5. Minimum Rebar Spacing for CIP Concrete (mm)

Bar Size	**Minimum Spacing (mm)**			
				
13	51	51	63	63
16	54	56	70	70
19	57	68	76	83
22	60	78	82	96
25	64	90	90	110
29	72	101	101	124
32	81	114	114	140
36	90	127	127	155
43	108	152	152	183
57	143	203	203	203

3. DESIGN EXAMPLE

- **Solid slab bridge design**

A simple span concrete slab bridge with clear span length (S) of 9150 mm is shown in **Figure 4.8**. The total width (W) is 10,700 mm, and the roadway is 9640 mm wide (W_R) with 75 mm (d_w) of future wearing surface. The material properties are as follows:

- Density of wearing surface ρ_w = 2250 kg/m^3.
- Concrete strength f'_c= 28 MPa.
- Concrete density ρ_c = 2400 kg/m^3.
- Concrete modulus of elasticity E_c= 26,750 MPa.

- Yield strength of reinforcement f_y= 420 MPa.
- Steel modulus of elasticity E_c= 200,000 MPa, n=8.

Requirements:

Design the slab reinforcement base on AASHTO-LRFD (1994) Strength I and Service I (cracks) Limit States.

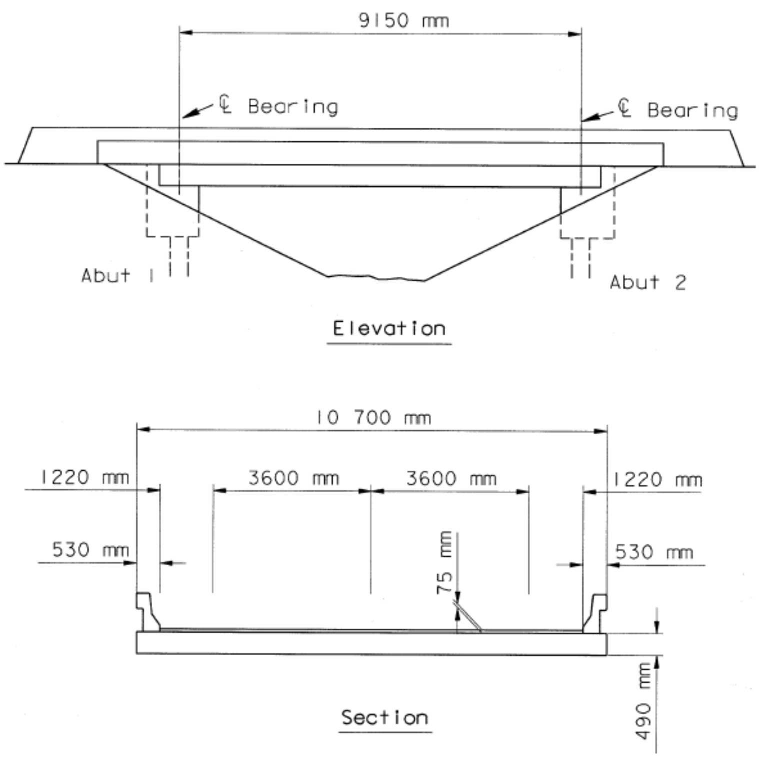


Fig. 5.8. Solid slab bridge design example.

3.1. Selection of Deck Thickness

The minimum deck thickness $h_{\min}$ is determined by the following expression:

$$h_{\min} = 1.2(\frac{S + 3000}{30})$$

Given:

$$S = 9150\,\text{mm}$$

$$h_{\min} = 1.2(\frac{9150 + 3000}{30}) = 486\,\text{mm}$$

Therefore, the selected deck thickness is:

$$\boxed{h = 490\ \text{mm}}$$

3.2. Determination of Live Load Equivalent Strip Width

(According to **AASHTO 4.6.2.3** and **4.6.2.1.4b**)

a. Interior Strip Width

i. Single-Lane Loading

The equivalent strip width for the interior girder is calculated using:

$$E_{\text{interior}} = 250 + 0.42\sqrt{L_1 W_1}$$

where:

- L_1: lesser of the actual span length or **18,000 mm**
- W_1: lesser of the actual deck width or **9,000 mm** (for single-lane loading), or **18,000 mm** (for multilane loading)

Substituting the known values:

$$L_1 = 9150\ \text{mm}, W_1 = 9000\ \text{mm}$$

$$E_{\text{interior}} = 250 + 0.42\sqrt{(9150)(9000)} = 4061\ \text{mm}$$

Hence, the **live load equivalent strip width** for the interior girder is:

$$\boxed{E_{\text{interior}} = 4060\ \text{mm}}$$

ii. Multilane Loading

The number of lanes is determined as:

$$N_L = \text{INT}(\frac{W}{3600}) = \text{INT}(\frac{10{,}700}{3600}) = 2$$

$$\frac{W}{N_L} = \frac{10{,}700}{2} = 5350\ \text{mm}$$

Then, the equivalent interior strip width is:

$$E_{\text{interior}} = 2100 + 0.12\sqrt{L_1 W_1}$$

$$E_{\text{interior}} = 2100 + 0.12\sqrt{(9150)(10{,}700)} = 3287\ \text{mm} < 5350\ \text{mm}$$

Hence:

$$\boxed{E_{\text{interior}} = 3287\ \text{mm}}$$

---

b. Edge Strip Width

The effective strip width near the edge is calculated as the distance between the edge of the deck and the inner face of the barrier, plus half the strip width:

$$E_{\text{edge}} = 530 + 300 + \frac{3287}{2} = 2324\ \text{mm} > 1800\ \text{mm}$$

Therefore, the adopted value is:

$$\boxed{E_{\text{edge}} = 1800\ \text{mm}}$$

3.3.Dead Load

a. Slab Self-Weight

$$W_{\text{slab}} = (0.49)(2400)(9.81)(10^{-3}) = 11.54\ \text{kN/m}^2$$

b. Future Wearing Surface

$$W_{\text{w}} = (0.075)(2250)(9.81)(10^{-3}) = 1.66\ \text{kN/m}^2$$

c. Concrete Barrier

Assume 0.24 m^3 of concrete per meter length of the barrier:

$$W_{\text{barrier}} = (0.24)(2400)(9.81)(10^{-3}) = 5.65\ \text{kN/m}^2$$

3.4.Calculation of Live-Load Moments

The moment at midspan controls the design.

a. Moment due to Design Truck (Fig. 4.8)

$$M_{\text{LL-Truck}} = (214.2)(4.575) - (145)(4.3) = 356.47\ \text{kN.m}$$

b. Moment due to Design Tandem (Fig. 4.9)

$$M_{\text{LL-Tandem}} = (95.58)(4.575) = 437.28\ \text{kN.m}$$

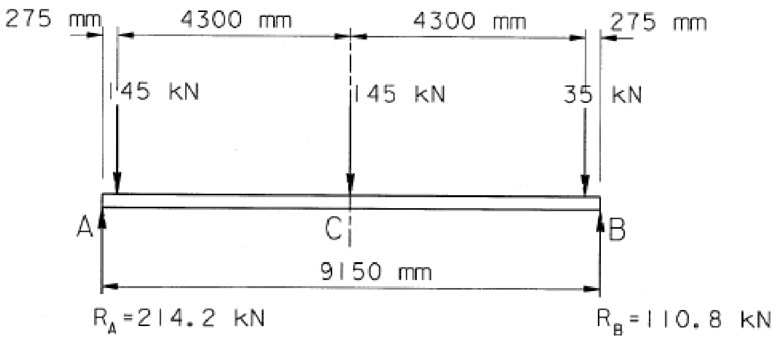


Fig. 5.9. Position of design truck for maximum moment.

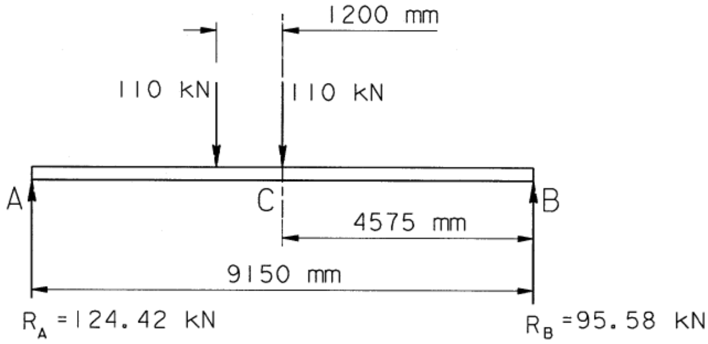


Fig. 5.10. Position of tandem for maximum moment.

c. Moment Due to Lane Load

For the lane load, the applied moment is:

$$M_{\text{LL-Lane}} = \frac{(9.3)(9.15)^2}{8} = 97.32 \text{ kN.m}$$

3.5.Determination of Load Factors and Load Combinations

(According to **AASHTO Table 3.4.1-1** and **Article 1.3.3-5**)

a. Strength I Limit State Load Factors

Weight of superstructure (DC)	: 1.25
Weight of wearing surface (DW)	: 1.50
Live Load (LL)	: 1.75

Adjustment factors:

$$\eta = \eta_D\, \eta_R\, \eta_I = (0.95)(1.05)(0.95) = 0.948 \leq 0.95$$

$$\boxed{\text{Use } \eta = 0.95}$$

---

b. Interior Strip Moment (1 m wide)

(According to **AASHTO 3.6.2.1** and **3.6.1.2.4**)

Dynamic Load Factor: $IM = 0.33$

Lane Load: $M_{\text{LL-Lane}} = \frac{97.32}{3.287} = 29.61 \text{ kN.m}$

Live Load: $M_{\text{LL+IM}} = (1 + 0.33)\frac{437.28}{3.287} + 29.61 = 206.54 \text{ kN.m}$

Future Wearing Surface: $M_{\text{DW}} = \frac{W_{\text{fw}}L^2}{8} = \frac{(1.66)(9.15)^2}{8} = 17.37 \text{ kN.m}$

Dead Load: $M_{\text{DC}} = \frac{W_{\text{slab}}L^2}{8} = \frac{(11.54)(9.15)^2}{8} = 120.77 \text{ kN.m}$

Factored Moment:

$$M_u = \eta[1.25(M_{\text{DC}}) + 1.50(M_{\text{DW}}) + 1.75(M_{\text{LL+IM}})]$$

$$M_u = (0.95)[1.25(120.77) + 1.50(17.37) + 1.75(206.54)] = \boxed{511.54 \text{ kN.m}}$$

---

c. Edge Strip Moment (1 m wide)

(According to **AASHTO Table 3.6.1.1.2-1**)

The end strip is limited to **half the lane width**. Use multiple presence factor = 1.2 and half design lane load.

Lane load: $M_{\text{LL-Lane}} = (1.2)(\frac{1}{2})\frac{97.3}{1.8} = 32.44$ kN.m

Live load: $M_{\text{LL+IM}} = (1 + 0.33)(1.2)(\frac{1}{2})\frac{437.28}{1.8} + 32.44 = 226.3$ kN.m

Dead load: $M_{\text{DC}} = \frac{(11.54+5.65)(9.15)^2}{8(1.8)} = 153.63$ kN.m

Future wearing: $M_{\text{DW}} = (1.66)(\frac{1.8-0.53}{1.8})\frac{(9.15)^2}{8} = 12.25$ kN.m

Factored Moment:

$$M_u = (0.95)[(1.25)(153.63) + (1.50)(12.25) + (1.75)(226.3)] = \boxed{579.12 \text{ kN.m}}$$

3.6.Reinforcement Design

a) Interior strip

Assume **No. 25 bars**.

Effective depth:

$$d = h - \text{cover} - \frac{\phi}{2} = 490 - 25 - \frac{25}{2} = 452.5 \text{ mm}$$

Design equations (neglect compression steel; take $b_w = b = 1000$mm):

$$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right), a = c\beta_1 = \frac{A_s f_y}{0.85 f_c' b_w}$$

Conveniently, use the reinforcement index

$$R_u = \frac{M_u}{\phi b d^2} = \frac{511.54 \times 10^6}{(0.9)(1000)(452.5)^2} = 2.766 \text{ N/mm}$$

Material ratio

$$m = \frac{f_y}{0.85 f_c'} = \frac{420}{0.85 \times 28} = 17.647$$

Steel ratio (tension-controlled section without compression flange):

$$\rho = \frac{1}{m}\left[1 - \sqrt{1 - \frac{2mR_u}{f_y}}\right] = \frac{1}{17.647}\left[1 - \sqrt{1 - \frac{2(17.647)(2.766)}{420}}\right] = 0.00705$$

Required steel area per meter width:

$$A_s = \rho\, b\, d = (0.00705)(1000)(452.5) = \boxed{3189\text{mm}^2/\text{m}}$$

Use **No. 25 bars @ 150 mm**:

Area provided per meter = $\frac{510}{0.150} \approx 3400\ \text{mm}^2/\text{m}$ ($\geq$ 3189 mm^2/m) ⇒ **OK**.

(510 mm^2 is the area of one No. 25 bar.)

---

i) Check section limits

For $f_c' = 28$MPa,

$$\beta_1 = 0.85$$

Neutral axis depth:

$$c = \frac{a}{\beta_1} = \frac{A_s f_y}{0.85\, \beta_1\, f_c'\, b_w} = \frac{(510)(420)}{0.85(0.85)(28)(150)} = 70.6\ \text{mm}$$

Ratio:

$$\frac{c}{d} = \frac{70.79}{452.5} = 0.156 \leq 0.42 \Rightarrow \text{**OK (tension-controlled)**}$$

Minimum Steel Ratio Check

Provided steel ratio (with No. 25 @ 150 mm):

$$\rho_{\text{prov}} = \frac{A_{\text{bar}}}{s\,d} = \frac{510}{(150)(452.5)} = 0.00751$$

Code minimum tension-steel ratio:

$$\rho_{\min} = 0.03(\frac{f_c'}{f_y}) = 0.03(\frac{28}{420}) = 0.00200$$

Comparison:

$$\rho_{\text{prov}} = 0.00751 \;\; \geq \;\; \rho_{\min} = 0.00200 \Rightarrow \text{OK}$$

Thus, the provided reinforcement satisfies the minimum reinforcement requirement.

ii. Crack Control

The **service load moment** is obtained as:

$$M_{sa} = 1.0[1.0(M_{DC}) + 1.0(M_{DW}) + 1.0(M_{LL+IM})]$$
$$M_{sa} = [120.77 + 17.37 + (176.93 + 29.61) = 344.68 \text{ kN.}$$

Cracking Check

Allowable tensile stress in concrete:

$$0.8f_r = 0.8(0.63\sqrt{f_c'}) = 0.8(0.63)\sqrt{28} = 2.66 \text{ MPa}$$

Tensile stress in concrete:

$$f_c = \frac{M_{sa}}{S} = \frac{344.68 \times 10^6}{(490)^2/6} = 8.61 \text{ MPa}$$

Since $f_c = 8.61 > 2.66$ MPa, **the section is cracked**.

Cracked Section Properties

$$n = 8, b = 150.0 \text{ mm}, A_s = 510\text{mm}^2, d = 452.5 \text{ mm}$$
$$B = \frac{1}{b}(nA_s) = \frac{1}{150}(8)(510) = 27.2$$
$$C = \frac{2}{b}(nA_s d) = \frac{2}{150}(8)(452.5)(510) = 24616$$

Neutral axis depth:

$$x = \sqrt{B^2 + C} - B = \sqrt{(27.2)^2 + 24616} - 27.2 = 132 \text{ mm}$$

Moment of inertia of cracked section:

$$I_{cr} = \frac{1}{3}bx^3 + nA_s(d - x)^2$$
$$I_{cr} = \frac{1}{3}(150)(132)^3 + (8)(510)(452.5 - 132)^2 = 534.1 \times 10^6 \text{ mm}^4$$

Stress in Steel

$$f_s = n\frac{M_{sa}(d - x)}{I_{cr}}$$
$$f_s = (8)\frac{(344.68)(452.5 - 132)}{534.1 \times 10^6} = 248 \text{ MPa}$$

Allowable Tensile Stress in Reinforcement

From Eq. (9.5):

$$f_{sa} = \frac{Z}{(d_c A_s)^{1/3}} \leq 0.6 f_y$$

Given:

$Z = 23{,}000$ N/mm (moderate exposure),

$d_c = 25 + \frac{25}{2} = 37.5$ mm,

bar spacing = 150 mm:

$$A_s = 2d_c \times \text{spacing} = 2(37.5)(150) = 11{,}250\text{mm}^2$$

$$f_{sa} = \frac{23{,}000}{[(37.5)(11{,}250)]^{1/3}} = 307 \text{ MPa}$$

$$0.6 f_y = 0.6(420) = 252 \text{ MPa}$$

Hence:

$$f_s = 248 \text{ MPa} \leq 252 \text{ MPa} \Rightarrow \text{OK}$$

✅ **Provide No. 25 bars @ 150 mm** for interior strip.

---

b) Edge Strip

By similar procedure:

Use No. 25 bars @ 125 mm.

3.7.Determine Distribution Reinforcement (AASHTO 5.14.4.1)

For positive moment regions, the **bottom transverse reinforcement** may be taken as a **percentage of the main longitudinal reinforcement**:

$$\frac{1750}{\sqrt{L}} \leq 50\% \Rightarrow \frac{1750}{\sqrt{9150}} = 18.3\% \leq 50\%$$

Thus, use **18.3%** of the main tension steel for transverse distribution reinforcement.

---

a) Interior strip

Main longitudinal reinforcement: No. 25 @ 150 mm

$$A_s = \frac{510}{150} = 3.40 \text{ mm}^2/\text{mm}$$

Required transverse steel:

$$A_{t,\text{req}} = 0.183 \times 3.40 = 0.622\text{mm}^2/\text{mm}$$

Provide: No. 16 @ 300 mm (transverse bottom bars)

$$A_t = \frac{199}{300} = 0.663\text{mm}^2/\text{mm} \quad (\geq 0.622) \Rightarrow \text{ OK}$$

---

b) End (edge) strip

Main longitudinal reinforcement: No. 25 @ 125 mm

$$A_s = \frac{510}{125} = 4.08 \text{ mm}^2/\text{mm}$$

Required transverse steel:

$$A_{t,\text{req}} = 0.183 \times 4.08 = 0.746\text{mm}^2/\text{mm}$$

Provide: No. 16 @ 250 mm

$$A_t = \frac{199}{250} = 0.796 \text{ mm}^2/\text{mm} \quad (\geq 0.746) \Rightarrow \text{ OK}$$

Construction simplification

For uniform detailing and ease of placement, adopt a single layout across the deck:

Use No. 16 @ 250 mm transverse (bottom) across the entire bridge width

---

3.8.Shrinkage and Temperature Reinforcement (AASHTO 5.10.8)

For the deck slab, the minimum shrinkage-and-temperature steel in **each direction** is:

$$A_s \geq 0.75 \frac{A_g}{f_y}$$

Per unit width of slab: $A_g = 1 \times h = 490 \text{ mm}^2/\text{mm}$and $f_y = 420$ MPa.

Thus,

$$A_s \geq 0.75 \frac{(1)(490)}{420} = 0.875\text{mm}^2/\text{mm(each direction)}$$

Top reinforcement is split into two layers, therefore per layer:

$$A_{s,\text{top layer}} = \frac{0.875}{2} = 0.438\text{mm}^2/\text{mm}$$

Provide: No. 13 @ 300 mm transverse top bars

$$A_s = \frac{129}{300} = 0.430 \text{ mm}^2/\text{mm} \quad \approx 0.438 \Rightarrow \text{ OK}$$

3.9.Summary / Notes to Designer

- To complete the design, verify **all limit states** (Strength, Service, Fatigue, Extreme Event) with the governing **AASHTO LRFD load combinations**.
- Check **long-term deflection**, cracking near supports, and distribution of reinforcement for longer or continuous spans.
- For **large skew bridges**, re-evaluate rebar layout and development to accommodate skew effects and ensure constructability.
- Final drawings should show consistent bar marks, splice lengths, cover, and the uniform transverse distribution steel selected (**No. 16 @ 250 mm bottom**, **No. 13 @ 300 mm top**).

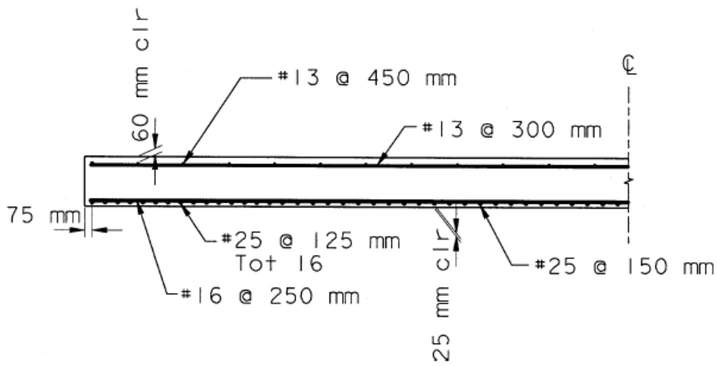


Fig. 5.11. Slab reinforcement detail.

References

1. Design Codes and Standards

1. CEN. (2004). *Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings* (EN 1992-1-1). Brussels: European Committee for Standardization.
2. CEN. (2004). *Eurocode 7: Geotechnical design – Part 1: General rules* (EN 1997-1). Brussels: CEN.
3. CEN. (2005). *Eurocode 2 – Part 3: Liquid retaining and containment structures* (EN 1992-3). Brussels: CEN.
4. CEN. (2006). *Eurocode 1: Actions on structures – Part 4: Silos and tanks* (EN 1991-4). Brussels: CEN.
5. ACI Committee 318. (2019). *Building code requirements for structural concrete (ACI 318-19)*. American Concrete Institute.
6. fib (CEB-FIP). (2010). *Model code for concrete structures 2010*. International Federation for Structural Concrete.
7. AASHTO. (2020). *LRFD bridge design specifications* (9th ed.). American Association of State Highway and Transportation Officials.
8. AASHTO. (1997). *Guide specifications for design and construction of segmental concrete bridges* (Draft 2nd ed.). American Association of State Highway and Transportation Officials.
9. ICC. (2021). *International building code – Chapter 31: Special construction*. https://codes.iccsafe.org/content/IBC2021P2/chapter-31-special-construction

2. General RC Design Textbooks

10. Thonier, H. (n.d.). *Conception et calcul des structures de bâtiment* (Tomes 1–6). Presses de l'École Nationale des Ponts et Chaussées.
11. Guerrein, A. (n.d.). *Traité de béton armé*. Dunod.
12. Nilson, A. H., Darwin, D., & Dolan, C. W. (2016). *Design of concrete structures* (15th ed.). McGraw-Hill Education.

13. Park, R., & Paulay, T. (1975). *Reinforced concrete structures*. John Wiley & Sons.

14. MacGregor, J. G., & Wight, J. K. (2014). *Reinforced concrete: Mechanics and design* (7th ed.). Pearson.

15. Mosley, W. H., Bungey, J. H., & Hulse, R. (2012). *Reinforced concrete design* (7th ed.). Palgrave Macmillan.

16. Ghali, A., Favre, R., & Elbadry, M. (2011). *Concrete structures: Stresses and deformations* (4th ed.). Taylor & Francis.

17. Reynolds, C. E., Steedman, J. C., & Threlfall, A. J. (2007). *Reinforced concrete designer's handbook* (11th ed.). Taylor & Francis.

18. Wang, C.-K., Salmon, C. G., & Pincheira, J. (2018). *Reinforced concrete design* (9th ed.). Wiley.

19. Hambly, E. C. (1991). *Bridge deck behaviour*. Taylor & Francis.

20. Leonhardt, F. (1982). *Prestressed concrete bridges*. Birkhäuser.

21. Podolny, W., & Muller, J. (1982). *Prestressed concrete segmental bridges*. Wiley.

22. Barker, R., & Puckett, J. (2013). *Design of highway bridges: An LRFD approach*. Wiley.

23. Ice Manual of Bridge Engineering (2019). *ICE manual of bridge engineering*. Thomas Telford.

24. Jenike, A. W. (1964). *Storage and flow of solids*. University of Utah.

25. Mendis, P., & Ngo, T. (2014). *Design of reinforced concrete tanks and silos*. Engineering Education Australia.

26. Jaeger, C. (1980). *Engineering design of concrete shells and domes*. American Concrete Institute.

27. Timoshenko, S., & Woinowsky-Krieger, S. (1959). *Theory of plates and shells*. McGraw-Hill.

28. Gupta, N. (2014). *Thin shell concrete structures: Analysis and design*. New Age International.

29. CIRIA. (2012). *Design of liquid retaining concrete structures* (CIRIA C703). Construction Industry Research and Information Association.

3. Chapter 01 — Retaining Walls

30. Tensar. (n.d.). *Types of retaining wall*. https://www.tensar.co.uk/resources/articles/types-of-retaining-wall

31. Soil Management India. (n.d.). *Types of retaining walls – 7 types*. https://www.soilmanagementindia.com/soil/retaining-walls/types-of-retaining-walls-7-types-soil-engineering/14637

32. DreamCivil. (n.d.). *Sheet piles*. https://dreamcivil.com/sheet-piles/

33. The Constructor. (n.d.). *Retaining wall – Types & uses*. https://theconstructor.org/geotechnical/retaining-wall-types-use/24566/

34. Structural Guide. (n.d.). *Stability calculation of retaining walls*. https://www.structuralguide.com/stability-calculation-of-retaining-walls/

35. The Structural World. (2019). *Design of retaining wall*. https://www.thestructuralworld.com/2019/03/04/design-of-retaining-wall/

36. CE-Ref. (n.d.). *RC retaining wall design*. http://www.ce-ref.com/Foundation/Retaining_wall/Ret_RC_design/RC_retain_wall.html

37. Bowles, J. E. (1997). *Foundation analysis and design*. McGraw-Hill.

38. Das, B. M. (2010). *Principles of foundation engineering* (7th ed.). Cengage Learning.

39. Teng, W. C. (1962). *Foundation design*. Prentice-Hall.

4. Chapter 02 — Domes

40. HotCore. (n.d.). *Kareff – 101579*. https://hotcore.info/act/kareff-101579.html

41. SlideShare. (n.d.). *Construction of domes*. https://www.slideshare.net/samikshachoudhary7731/construction-of-domes

42. Tureng. (n.d.). *Arc surbaissé – Translation*. https://tureng.com/fr/francais-anglais/arc%20surbaiss%C3%A9

43. Gupta, N. (2014). *Thin shell concrete structures*. New Age Publications.

5. Chapter 03 — Water Tanks & Towers

44. BYJU's Exam Prep. (n.d.). *Types of loads*. https://byjusexamprep.com/gate-ce/types-of-loads#toc-2

45. IndiaMART. (n.d.). *Square concrete water tank*. https://www.indiamart.com/proddetail/square-concrete-water-tank-17358811291.html

46. Shay Murtagh. (n.d.). *Concrete storage tanks*. https://www.shaymurtagh.co.uk/precast-concrete-products/concrete-storage-tanks/

47. Panthers Concrete Tanks. (n.d.). *Residential tanks*. https://www.pantherconcretetanks.com.au/residential-tanks/

48. Binguimatchi, D. B. (n.d.). *Master thesis*. Groupes des écoles ETER–ETSHER.

49. USACE. (1992). *Concrete water tanks* (EM 1110-2-2002). U.S. Army Corps of Engineers.

6. Chapter 04 — Silos

50. fib Bulletin 72. (2014). *Design of circular and rectangular concrete silos*. Fédération Internationale du Béton.

51. Study.com. (n.d.). *Silos in architecture, design & structure*. https://study.com/academy/lesson/silos-in-architecture-design-structure.html

52. Walker, D. M. (1966). *Pressures and forces in silos*. *Chemical Engineering Science*, 21(11), 975–997.

53. Mendis, P., & Ngo, T. (2014). *Design of reinforced concrete tanks and silos*. Engineering Education Australia.

7. Chapter 05 — Reinforced Concrete Bridges

54. Lyang, J., Lee, D., & Kung, J. (2000). Reinforced concrete bridges. In W.-F. Chen & L. Duan (Eds.), *Bridge engineering handbook*. CRC Press.

55. FHWA. (n.d.). *This bridge*. https://www.environment.fhwa.dot.gov/Env_topics/historic_pres/post1945_engineering/this_bridge.aspx

56. Midasoft. (n.d.). *Structural evaluation of concrete slab/T-beam bridges*. https://www.midasoft.com/bridge-library/structural-evaluation-of-concrete-slab/t-beam-bridges

57. The Constructor. (n.d.). *Construction of box girder bridge*. https://theconstructor.org/structures/construction-box-girder-bridge/2166/

58. ICE. (2019). *Manual of bridge engineering* (2nd ed.). Thomas Telford.