

Ministry of Higher Education and Scientific Research
Tissemsilt University



Faculty of Science and Technology
Department of Science and Technology

Composite materials

By
Dr. ADIM Belkacem

Foreword

This handout is intended for second-year students of the LMD Master's degree in Materials in Civil Engineering and engineering students.

Based on the rich documentation available in the field, we have developed this work, presented as a support for the composite materials course. The course handout defines the composite materials as well as their different types. It illustrates the basics of composites as an essential discipline in the field of material engineering, where composites are considered as a key material in developing a strong, light and robust advanced structures. By then defining the material's different components and manufacturing processes, after that we determine its physical and mechanical characteristics as well as its mechanical behaviour under internal and external loads.

In addition to addressing the majority of materials identification and categorization, this handout serves as a checklist for students, practitioners, designers, manufacturers, and inspectors to quickly understand terminology and formulae helpful for basic applications.

The interested person is then encouraged to review the suggested bibliographic list at the conclusion of this document in order to further expand their understanding of the various ideas. Among the document's immediate advantages will be its instructional and scientific benefits.

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Chapter 1: Generalities

1.1. Introduction

Composite materials are materials with high mechanical performance, which can be shaped at will according to the designer's wishes and therefore have unlimited potential. Composite materials are developing today in practically all fields and are at the origin of formidable challenges in various high-tech achievements.

Thus, the development of the use of composite materials in structures requires the implementation of the necessary tools for modelling the mechanical behaviour of composite materials and analysing laminated or sandwich structures.

1.2. History

The first uses of composites date back to the 1500s B.C. when early Egyptians and Mesopotamian settlers used a mixture of mud and straw to create strong and durable buildings. Straw continued to provide reinforcement to ancient composite products including pottery and boats. Later, in 1200 AD, the Mongols invented the first composite bow. Using a combination of wood, bone, and “animal glue,” bows were pressed and wrapped with birch bark. These bows were extremely powerful and extremely accurate. Composite Mongolian bows provided Genghis Khan with military dominance, and because of the composite technology, this weapon was the most powerful weapon on earth until the invention of gunpowder.

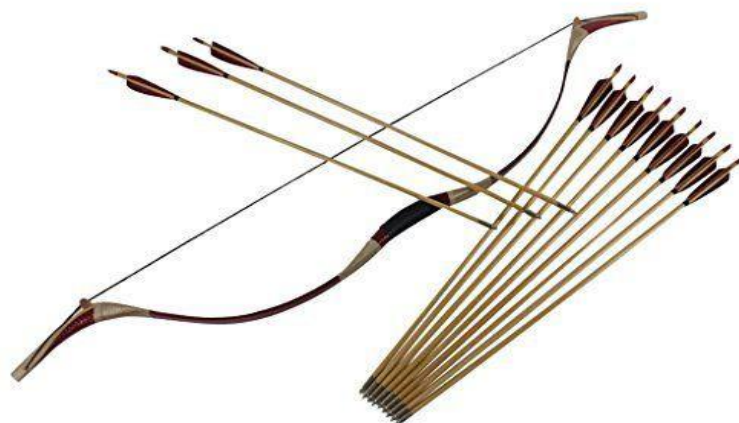


Figure 1.1: Composite Mongolian bow.

The modern era of composites did not begin until scientists developed plastics. Until then, natural resins derived from plants and animals were the only source of glues and binders. In the early 1900s, plastics such as vinyl, polystyrene, phenolic and polyester were developed. These new synthetic materials outperformed resins that were derived from nature. However, plastics alone could not provide enough strength for structural applications. Reinforcement was needed to provide the strength, and rigidity. In 1935, Owens Corning introduced the first glass fiber, fiberglass. Fiberglass, when combined with a plastic polymer creates an incredibly strong structure that is also lightweight. This is the beginning of the Fiber Reinforced Polymers (FRP) industry as we know it today.

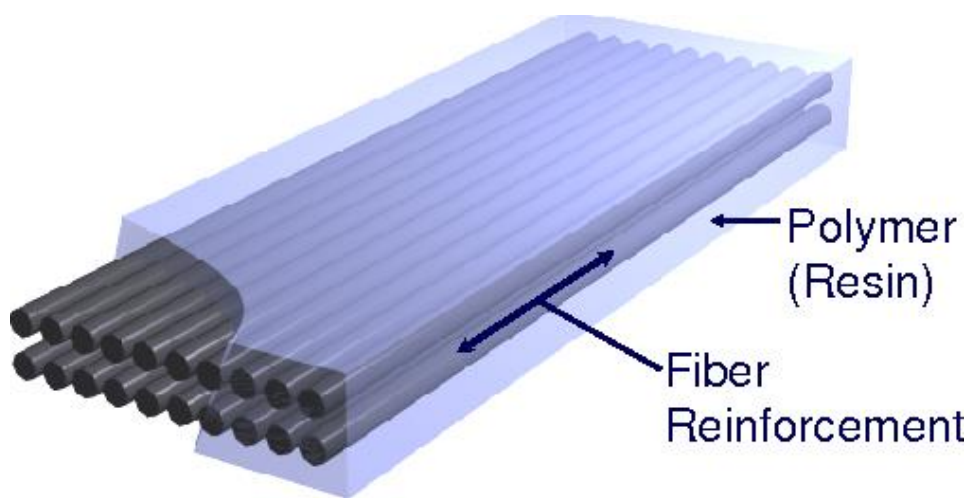


Figure 1.2: Fiber Reinforced Polymers (FRP).

WWII - Driving Early Composites Innovation:

Many of the greatest advancements in composites were incubated by war. Just as the Mongols developed the composite bow, World War II brought the FRP industry from the laboratory into actual production. Alternative materials were needed for lightweight applications in military aircraft. Engineers soon realized other benefits of composites beyond being lightweight and strong. It was discovered that fiberglass composites were transparent to radio frequencies, and the material was soon adapted for use in sheltering electronic radar equipment (Radomes).



Figure 1.3: Fiber Reinforced Polymers (FRP).

Adapting Composites: “Space Age” to “Everyday”:

By the end of the WWII, a small niche composites industry was in full swing. With lower demand for military products, the few composites’ innovators were now ambitiously trying to introduce composites into other markets. Boats were an obvious fit for composites, and the first commercial boat hull was introduced in 1946. At this time Brandt Goldsworthy, often referred to as the “grandfather of composites,” developed new manufacturing processes and products. He is credited with numerous advancements including being the first to fiberglass a surfboard, which revolutionized the sport. Goldsworthy also invented a manufacturing process known as pultrusion. Today, products manufactured from this process include ladder rails, tool handles, pipes, arrow shafts, Armor, train floors, medical devices, and more.

Continued Advancement in Composites:

In the 1970s the composites industry began to mature. Better plastic resins and improved reinforcing fibers were developed. DuPont developed an aramid fiber known as Kevlar; this fiber has become the standard in Armor due to its high tenacity. Carbon fibers was also developed around this time; it has since been replacing metal as the new material of choice. The composite industry is still evolving, with much of the growth is now focused around Renewable energy Wind turbine blades are constantly pushing the limits on size and are requiring advanced materials, designs, and manufacturing. In contrast to metallic alloys, each

material retains its separate chemical, physical, and mechanical properties. The two constituents are a reinforcement and a matrix.

1.3. Definition

The word composite in the term composite material signifies that two or more materials are combined on a macroscopic scale to form a useful third material. The key is the macroscopic examination of a material wherein the components can be identified by the naked eye. Different materials can be combined on a microscopic scale, such as in alloying of metals, but the resulting material is, for all practical purposes, macroscopically homogeneous, i.e., the components cannot be distinguished by the naked eye and essentially act together. The advantage of composite materials is that, if well designed, they usually exhibit the best qualities of their components or constituents and often some qualities that neither constituent possesses. Some of the properties that can be improved by forming a composite material are:

- ✓ Strength
- ✓ Fatigue life
- ✓ Stiffness
- ✓ Temperature-dependent behaviour
- ✓ Corrosion resistance
- ✓ Thermal insulation
- ✓ Wear resistance
- ✓ Thermal conductivity
- ✓ Attractiveness
- ✓ Acoustical insulation
- ✓ Weight

Naturally, not all of these properties are improved at the same time nor is there usually any requirement to do so. In fact, some of the properties are in conflict with one another, e.g., thermal insulation versus thermal conductivity. The objective is merely to create a material that has only the characteristics needed to perform the design task.

Composite materials have a long history of usage. Their precise beginnings are unknown, but all recorded history contains references to some form of composite material. For example, Plywood was used by the ancient Egyptians when they realized that wood could be rearranged

to achieve superior strength and resistance to thermal expansion as well as to swelling caused by the absorption of moisture. Medieval swords and armor were constructed with layers of different metals. More recently, fiber-reinforced, resin-matrix composite materials that have high strength-to-weight and stiffness-to-weight ratios have become important in weight sensitive applications such as aircraft and space vehicles.

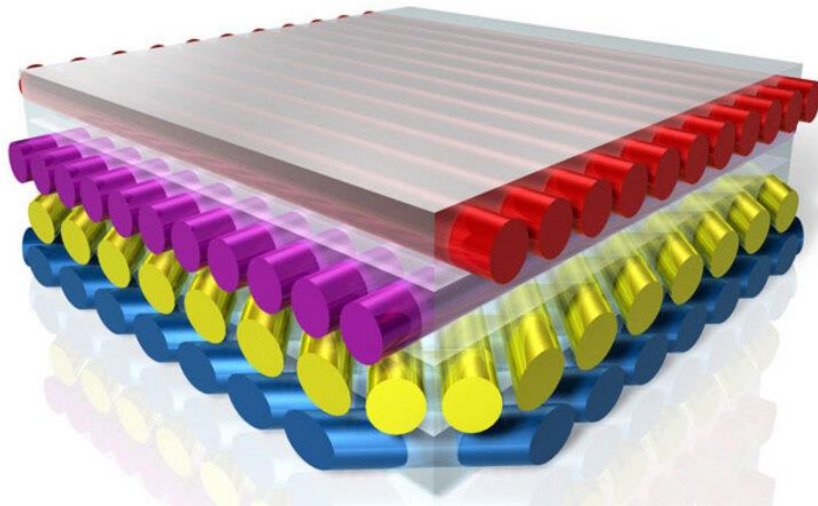


Figure 1.4: Example of composite material

Examples of naturally occurring composites:

- ✓ Wood: Cellulose fibers bound by lignin matrix
- ✓ Bone: Stiff mineral “fibers” in a soft organic matrix permeated with holes filled with liquids
- ✓ Granite: Granular composite of quartz, feldspar, and mica

Some examples of man-made composites:

- ✓ Concrete: Particulate composite of aggregates (limestone or granite), sand, cement and water
- ✓ Plywood: Several layers of wood veneer glued together
- ✓ Fiberglass: Plastic matrix reinforced by glass fibers
- ✓ Fibrous composites: Variety of fibers (glass, Kevlar, graphite, nylon, etc.) bound together by a polymeric matrix

These are not composites:

- ✓ Plastics: Even though they may have several “fillers”, their presence does not alter the physical properties significantly.
- ✓ Alloys: Here the alloy is not macroscopically heterogeneous, especially in terms of physical properties.
- ✓ Metals with impurities: The presence of impurities does not significantly alter physical properties of the metal

1.4. Classification of composite materials

Composites can be classified according to the shape (form) of the components or according to the nature of the components.

1.4.1. Classification according to the shape (form) of the constituents

Depending on the shape of the constituents, composites are classified into two main classes: particle composite materials and fiber composite materials.

1.4.1.1. Fiber composites

A composite material is a fiber composite if the reinforcement is in the form of fibers. The fibers used are either in the form of continuous fibers or in the form of discontinuous fibers: chopped fibers, short fibers, etc. The arrangement of the fibers and their orientation make it possible to modulate the mechanical properties of composite materials to obtain materials ranging from highly anisotropic materials to isotropic materials in a plane.

The designer therefore has a type of material whose mechanical and physical behaviours he can modify and modulate at will by playing on:

- ✓ The nature of the constituents,
- ✓ The proportion of the constituents,
- ✓ The orientation of the fibers,

according to the imposed specifications.

The importance of fiber composite materials justifies an exhaustive study of their mechanical behaviours. Consequently, this work will essentially be devoted to the study of this type of material.

1.4.1.2. Particle composites

A composite material is a particulate composite when the reinforcement is in the form of particles. A particle, as opposed to fibers, does not have a preferred dimension.

Particles are generally used to improve certain properties of materials or matrices, such as stiffness, temperature resistance, abrasion resistance, shrinkage reduction, etc. In many cases, particles are simply used as fillers to reduce the cost of the material, without reducing its characteristics.

The choice of matrix-particle combination depends on the desired properties. For example, lead inclusions in copper alloys will increase their ease of machining. Particles of brittle metals such as tungsten, chromium, and molybdenum, incorporated into ductile metals, will increase their properties at elevated temperatures, while retaining the ductile character at room temperature.

Cermets are also examples of particulate metal-ceramic composites, suitable for use at elevated temperatures. For example, oxide-based cermets are used for high-speed cutting tools and for high-temperature protection.

Also, elastomer particles can be incorporated into fragile polymer matrices to improve their fracture and impact properties by reducing their sensitivity to cracking.

Thus, particle composites cover a wide field whose development is constantly increasing.

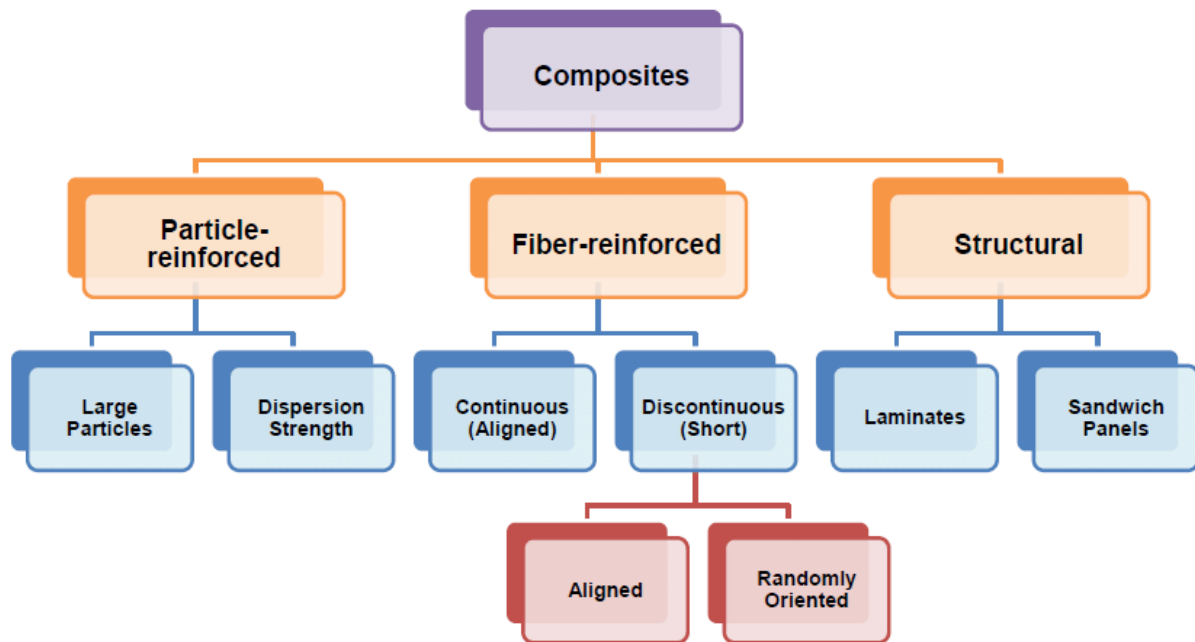


Figure 1.5: Classification according to the shape of the constituents

1.4.2. Classification according to the nature of the constituents

Depending on the nature of the matrix, composite materials are classified as organic matrix composites, metal matrix composites or mineral matrix composites.

Various reinforcements are associated with these matrices. Only certain pairs of associations currently have industrial use, others are being developed in research laboratories. Among these composites, we can mention:

1. Organic matrix composites (resin, fillers), with:
 - ✓ mineral fibres: glass, carbon, etc.
 - ✓ organic fibres: Kevlar, polyamides, etc.
 - ✓ metal fibres: boron, aluminium, etc.

2. Metal matrix composites (light and ultra-light alloys of aluminium, magnesium, titanium), with:
 - ✓ mineral fibres: carbon, silicon carbide (SiC),
 - ✓ metal fibres: boron,
 - ✓ metallo-mineral fibres: boron fibres coated with silicon carbide (BorSiC).

3. Mineral matrix composites (ceramic), with:

- ✓ metal fibres: boron,
- ✓ metal particles: cermets,
- ✓ mineral particles: carbides, nitrides, etc.

Organic matrix composite materials can only be used in the temperature range not exceeding 200 to 300 °C, while metallic or mineral matrix composite materials are used beyond this: up to 600 °C for a metallic matrix, up to 1000 °C for a ceramic matrix.

1.5. Application areas of composites

Composites are widely used in industry because they have good characteristics such as rigidity, lightness and resistance to mechanical and thermal stresses.

The choice of materials, such as in the aeronautics, aerospace, automotive, railway, sports and leisure, building, civil engineering is always a complex problem where cost/performance behaviour reigns.

1.5.1. Aeronautics sector

High-performance composites have long been confined in civil aeronautics to secondary structural parts (interior trim, flaps), more rarely primary and never to the wing, because two difficulties arose:

- ✓ High costs compared to aluminium,
- ✓ Risks of delamination prohibiting flight on loaded aerodynamic structures.

The first composite parts were introduced on satellites in the late 1970s. The materials used are mainly high Young's modulus carbon fibers with epoxy resin. The first composite materials were introduced on ballistic missiles in the mid-1960s using glass fiber/epoxy resin wound structures, with aramid fiber/epoxy wound structures being introduced in the late 1970s. Carbon fibers for the production of wound structures were introduced in the mid-1970s.

The use of composite materials in aeronautical construction results in substantial weight savings. Their use allows, for equal performance, weight savings ranging from 10% to 20%.

The weight of the medium-haul Boeing 767 aircraft was reduced, through the use of composites, by 921 kilograms compared to the conventional solution. Composite materials are mainly used for the design and production of powder boosters for satellite launchers and their propulsion gas ejection nozzle, obtained by winding glass, Kevlar or carbon fibers, in order to withstand the internal pressure resulting from the combustion of the carbon propellant, in order to withstand the internal pressure resulting from the combustion of the solid propellant and the launch forces which, in certain cases, result in shocks and vibrations.

On commercial aircraft, the first mass-produced aircraft mainly used aluminium alloys, steel and titanium, today; the most recent commercial aircraft still use 54% aluminium alloy in weight, but 20% composite materials have been introduced into the structure, 13% steel, 6% titanium and 7% miscellaneous materials.

Some obstacles to the mass use of composites are gradually being removed, particularly by foreign players, for example:

- ✓ As part of the Advanced Composite Technology program led by NASA and Boeing, a new process was developed to develop composite wings that are 20% cheaper than with conventional materials,
- ✓ Development of shape memory composites led by British Aerospace and Daimler Benz,
- ✓ Airbus' decision to equip the fuselages of its future A380s with composite technology called "Glare11111".

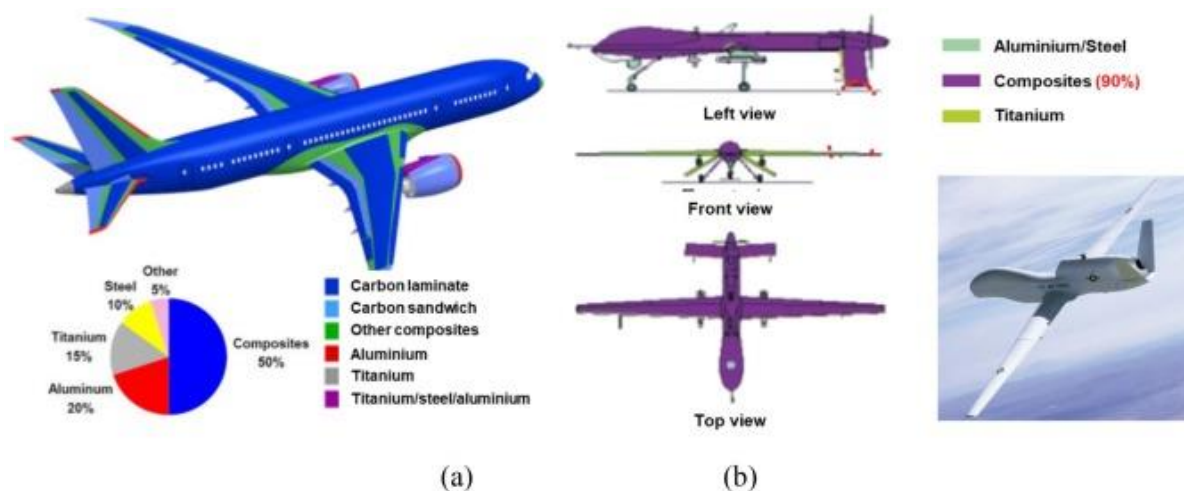


Figure 1.6: Applications of composites structures in aerospace industry (a) commercial aircrafts (Boeing 787), (b) unmanned aerial vehicles (MQ-1 predator)

1.5.2. Automotive sector

The automotive sector uses 95% polyester and glass fiber matrix composites, but has been massively integrating thermoplastic composites since 2003.

Composites provide the automotive sector with ease of maintenance and great freedom of design; the 1 kg weight reduction obtained by using composites allows an additional cost of 3€/kg gained or saves 1.5 € in fuel per 100 km.

In the automotive sector, the long-term development of composites is threatened by the difficulty of integrating them into manufacturing chains and by recyclability requirements. In Europe, approximately 12 million vehicles, with an average weight of 1000 kg are produced annually, or 12 million tons of materials.

While at its beginnings, approximately 100 years ago, an automobile was mainly made of wood and steel, today it brings together many materials belonging to different families:

- ✓ Ferrous materials: cast iron, steel (approximately 70% of its weight);
- ✓ Non-ferrous materials: aluminium, copper, magnesium (approximately 5%);
- ✓ Mineral materials: glass, ceramics (approximately 4%);
- ✓ Organic materials: paints, adhesives, textiles, fluids, rubbers, thermoplastics and thermosetting materials reinforced or not by fibers (glass, carbon, aramid, natural) or mineral fillers (approximately 20%).

Organic composite materials with a thermoplastic or thermosetting matrix reinforced with fibers, generally glass, short or long, appeared in the automobile industry during the 1960s and 1970s. Even if they are now used almost exclusively to fulfil certain functions, their rate of use does not exceed 10 to 15% depending on the vehicle.

A priori, these materials have three major handicaps compared to metallic materials in the context of use in the automobile sector:

- ✓ A high price per kilogram;
- ✓ Weaker mechanical characteristics;
- ✓ Often slow implementation processes with the exception of the injection process.

However, organic matrix composite materials have significant advantages:

- ✓ Low density;
- ✓ Molding implementation technologies which limit the material used in the sector, offer the possibility of obtaining parts with complex shapes and eliminate finishing machining.

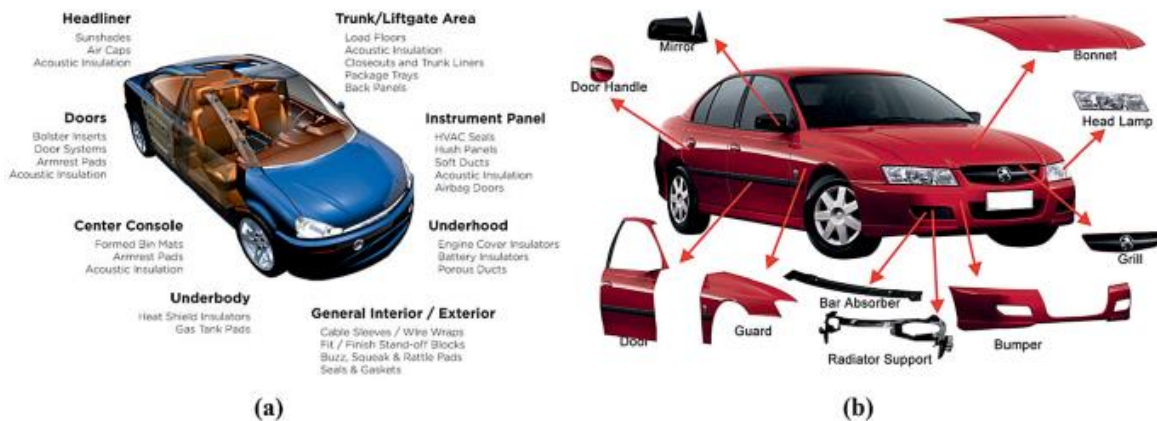


Figure 1.7: Application of composites in cars manufacturing

1.5.3. Construction sector

Apart from concrete, composites have relatively little penetrated the construction sector, in particular due to:

- ✓ A high price compared to traditional materials (6€/kg compared to 3€/kg for steel and 1€/kg for wood),
- ✓ A relative lack of knowledge of composites among architects and design offices,
- ✓ A lack of information on the life cycle (recyclability, durability).

The qualities of composites (lightness, fire resistance, mechanical and chemical resistance) allow us to predict a promising development. Composite solution suppliers (Acome, Uniject, Gimm Menuiseries, Owens Corning, etc.) have developed many products presented at the JEC trade fair in April 2000:

- ✓ Windows,
- ✓ Doors,
- ✓ Slate substitutes,

✓ Beam reinforcements



Figure 1.8: Application of composites in construction

1.5.4. Leisure and sports sectors

Thanks to their lightness, their good resistance to static and dynamic fatigue and their dimensional stability, composite materials are ideal materials for the manufacture and design of many sports and leisure items such as tennis rackets, fishing rods and hockey rods.

Manufacturers of sports and leisure equipment have long adopted composites, developed in particular by Shappe Technique and Ems Chimie (Switzerland); these new materials have allowed them to demonstrate great creativity, while presenting interesting mechanical properties which have, to a large extent, been at the origin of the recent development of modern board sports. (moulded and non-laminated skis).



Figure 1.9: Examples of application of composites in sports

1.5.5. Electrical sector

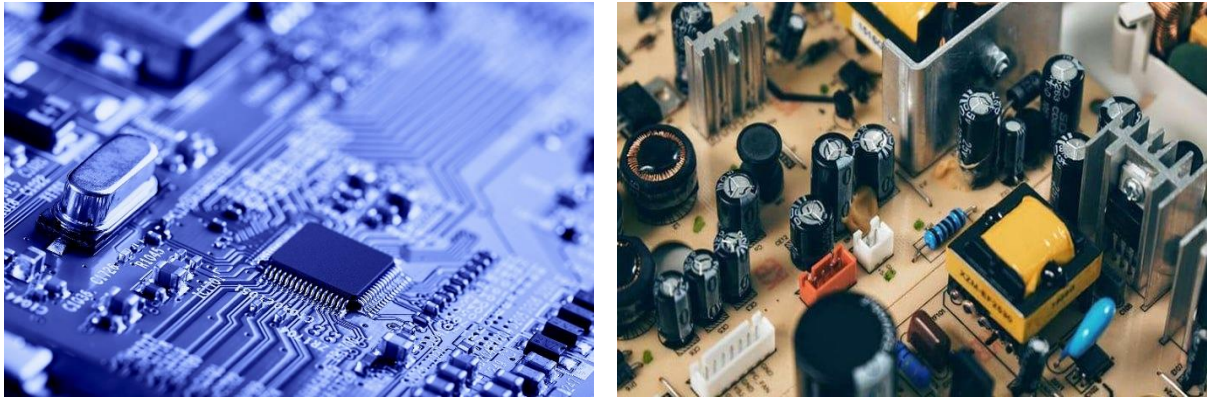


Figure 1.10: Examples of application of composites in Electronics

The massive consumption of GD composites, in the form of compressed SMC semi-finished products, in the electrical and electronic fields, is growing with the significant use of audiovisual (TV, microcomputers, DVD players, etc.).

By using the remarkable properties of composite materials, electrical construction produces reliable equipment with multiple functions and a long service life.

The production rates of electrical components, which can reach several thousand per day, are not very compatible with those of the SMC compression process. Increasing requirements for product recyclability constitute a significant threat to this sector of application of composites.

1.5.6. Industrial sector

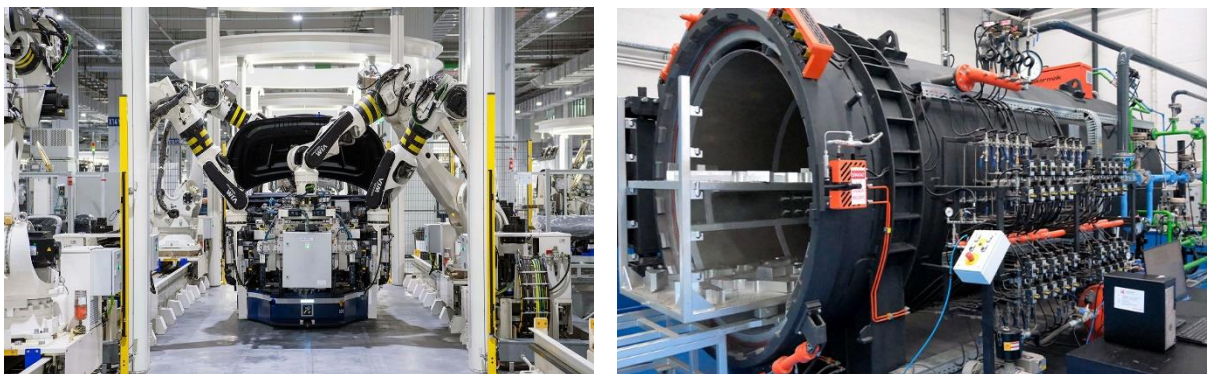


Figure 1.11: Composites in Industrial sector

The development of TD and TP composites, with the possibility of playing on their characteristics, have made these materials established in the industrial world. Carbon fibers can be used in addition to glass fibers both as reinforcement and as a sensor, and will be associated with polyester or epoxy resins.

The use of composite materials can contribute to improving the safety of certain sensitive industrial sites and facilitate the design of buildings.

For standard industrial constructions, composite materials still have the disadvantage of high cost compared to conventional materials, metals. Their characterization in a corrosive environment is still insufficiently known.

1.5.7. Railway sector



Figure 1.12: Composites in Railway sector

Composite materials are used in the railway sector for two main advantages: good fire behaviour and easy moulding of the parts used. The railway sector mainly uses composites with thermosetting matrix and long fibers, but it appears to be not very dynamic.

The generalization of composites in the railway sector is handicapped by their cost higher than 9€/kg, compared to 2€/kg for steel solutions and by the difficulties of the functional approach encountered by designers.

1.5.8. Medical sector

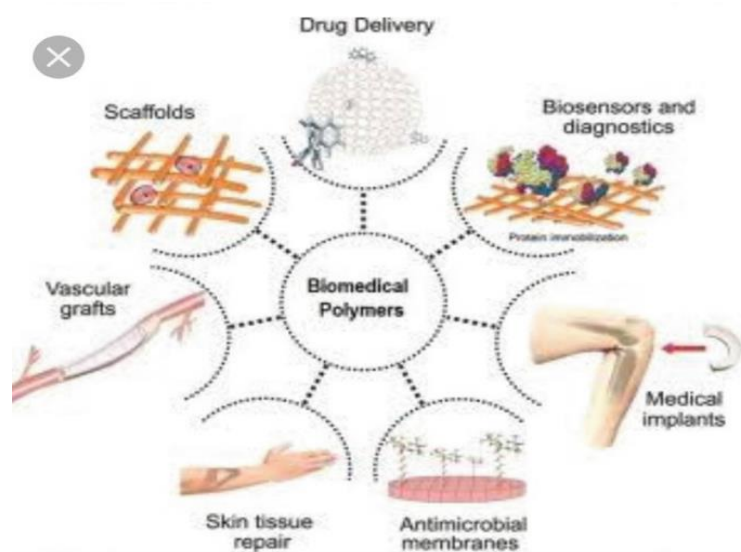


Figure 1.13: Composites in Medical sector

TD and TP composites have recently invested in the medical and security sector, as they previously invested in other sectors. These are mainly high-performance composites, with epoxy matrix and carbon fibers, or with thermoplastic matrix and long glass or carbon fibers on the other hand. The use of composites makes it possible to increase the reliability and precision of instruments thanks to better shock resistance, high rigidity and good resistance to corrosion (prostheses, stretchers).

The medical sector remains attached to the use of special steels whose price of €9/kg remains well below that of HP composites which can reach €50/kg.

1.5.9. Pathways to sustainable development

Composites are expected to develop in the coming years. However, to achieve this, the composite materials industry will have to integrate the constraints of sustainable development.

Thus, from 2004, 85% of the weight of end-of-life vehicles will have to be recycled. Currently, there is no fully operational and economically viable technical solution for recycling composites used in the automotive sector. These composites are, for the most part, thermosetting.

This major challenge for the sector depends on the mobilization of all stakeholders and their research and development efforts. A few industrial initiatives are developing in Europe, in particular Mecelec Composites et Recyclage in France, operator of the only composite recycling plant, and Ercom in Germany.

Technically, the recovery of thermosetting materials can involve grinding and using the aggregates obtained in the form of very low added value fillers for resins, bitumen and cements. Incineration in cement plants is a second possible option. However, recycling requirements should favour the use of thermoplastic composites, which can be more easily reprocessed.

In the nautical industry, personnel are exposed to the emissions of Volatile Organic Compounds (VOCs, particularly styrene). In Europe, Scandinavian and German industries apply stricter standards than in France and are campaigning for the harmonisation of national regulations. The strengthening of the standards currently in force in France would constitute an additional constraint affecting their costs for open-mould transformers.

In order for design offices to make greater use of composite materials, they will need to be better characterised in the future. This will enable designers to develop a functional analysis that integrates all their contributions. Indeed, these materials suffer from a lack of modelling and standardisation compared to traditional materials such as steel, which are subject to national or European standards. To effectively characterize these materials, design and modelling software must be developed more extensively than is currently the case.

1.6. Advantages and disadvantages of composites

Composites are preferred to other materials because they offer advantages related to:

- ✓ Their lightness;
- ✓ Their resistance to corrosion and also to fatigue;
- ✓ Their insensitivity to products such as greases, hydraulic fluids, paints and solvents;
- ✓ Their ability to take several forms, to integrate accessories and to allow noise reduction.

However, certain disadvantages hinder their diffusion:

- ✓ The costs of raw materials and manufacturing processes;
- ✓ The management of waste generated and increasingly strict regulations.

The composite materials industry must therefore today take up certain challenges such as:

- ✓ Controlling the emissions of volatile organic products, for example styrene;
- ✓ Controlling the transformation processes and performance of materials which implies a very good knowledge of the constituents put in place;
- ✓ The implementation of technologies and channels for end-of-life waste management, which is the most difficult part to satisfy due to the thermostable nature of most composites.

In addition, in this last point, recycling is currently very limited because grinding or other processes are very expensive and provide low-quality material. These few drawbacks and challenges to be overcome have pushed researchers and public authorities to invest and direct research towards renewable and biodegradable resources.

Chapter 2: Composite composition

2.1. Introduction

A composite material consists of a matrix and a reinforcement, made of fibers.

The matrix is itself composed of a resin (polyester, epoxy, etc.) and fillers whose aim is to improve the characteristics of the resin while reducing the production cost. From a mechanical point of view, the resin-filler assembly behaves like a homogeneous material, and the composite is considered to consist of a matrix and a reinforcement.

The reinforcement gives the composite material its high mechanical performance, while the matrix has the role of transmitting external mechanical stresses to the fibers and protecting the fibers from external aggressions.

The type of matrix-reinforcement association depends on the constraints imposed on the designer: high mechanical characteristics, temperature resistance, cost, corrosion resistance, etc.

The purpose of this chapter is to provide a general but non-exhaustive summary of the various constituents

2.2. Fibers and fabrics

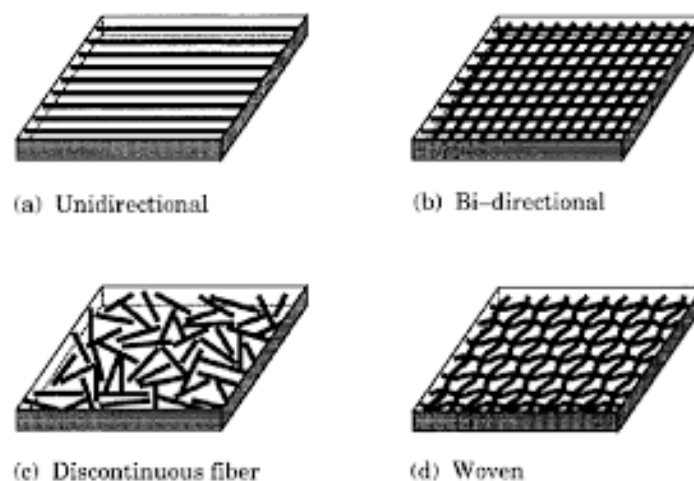


Figure 2.1: Different types of reinforcements

Reinforcement materials give composites their mechanical characteristics: rigidity, breaking strength, hardness, etc. These reinforcements also improve some of the physical properties: thermal behaviour, temperature resistance, fire resistance, abrasion resistance,

electrical properties, etc. The characteristics sought for reinforcements are: high mechanical characteristics, low density, good compatibility with resins, good ease of implementation, low cost, etc. Depending on the uses, reinforcements can be of various origins: plant, mineral, artificial, synthetic, etc. However, the most commonly used reinforcements are in the form of fibers or derived forms, and constitute a volume fraction of the composite material generally between 0.3 and 0.7. Fiber reinforcements come in various commercial forms:

- ✓ in linear form (threads, roving, etc.),
- ✓ in surface fabric form (simple fabrics, mats, etc.),
- ✓ in multidirectional form (braids, complex fabrics, etc.).

2.2.1. Linear forms



Figure 2.2: Linear glass fiber

The fibers are produced with a diameter of a few micrometres (around ten), and therefore cannot be used in unit form. For their practical use, these fibers are gathered into threads or strands of different shapes. The general nomenclature of the various linear forms is still poorly defined, and generally derived from that used for glass fibers. The unit fiber is generally called elementary filament or monofilament. The monofilaments are then gathered into threads or strands. Continuous or discontinuous threads are characterized by their linear weight or titre. This linear weight is a measure of the fineness of the threads, and it depends on the diameter and the number of monofilaments. The unit of linear weight is the tex, weight of a thread of length equal to 1000 m. That is:

$$1 \text{ tex} = 1 \text{ g/km}$$

In fact, we should rather write:

$$1 \text{ tex} = 10^{-6} \text{ kg/m}$$

In accordance with the S.I. system of units. The first definition, however, is better suited to practical use.

2.2.2. Surface forms

The yarns can be used to make surface shapes of various types: mats, fabrics or ribbons, mainly developed in the case of glass fibers.

2.2.2.1. Mats

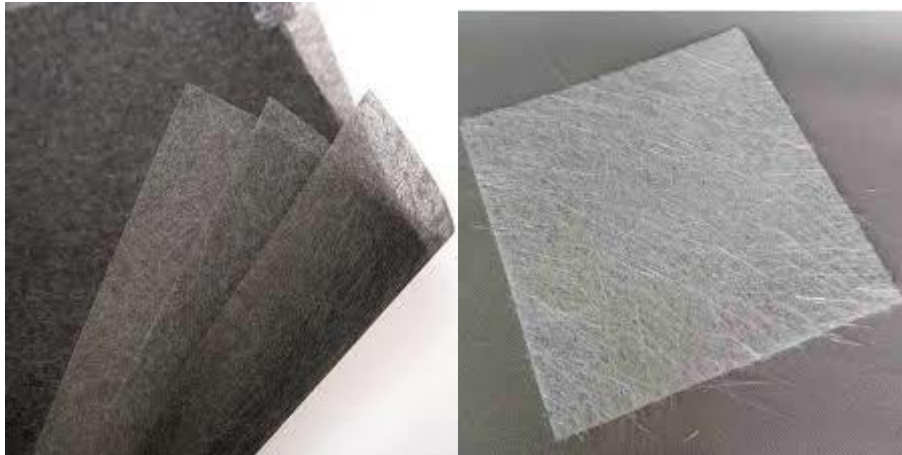


Figure 2.3: Example of mats

Mats are layers of continuous or discontinuous threads, arranged in a plane without any preferential orientation. They are held together by a binder which may or may not be soluble in the resins, depending on the application. The absence of preferential orientation of the fibers leads to an isotropy of the mechanical properties of the mat in its plane.

The difference between cut-strand mats and continuous-strand mats is essentially in their deformability property. The former are not very deformable while the latter allow deep “drawings” to be obtained by a regular elongation of the mat in all directions. One of the main applications of continuous-strand mats is the moulding between mould and counter-mould, of parts that can have complex shapes, by compression, injection or vacuum moulding.

2.2.2.2. Fabrics and ribbons



Figure 2.4: Different types of fabrics



Figure 2.5: Carbon ribbon

A fabric (or ribbon) is a surface assembly of threads, rovings, etc., made on a loom. It consists of (figure 2.6):

- ✓ a warp, a set of parallel threads distributed in a plane following the length of the fabric,
- ✓ of a weft, a set of threads intertwining with the warp threads.

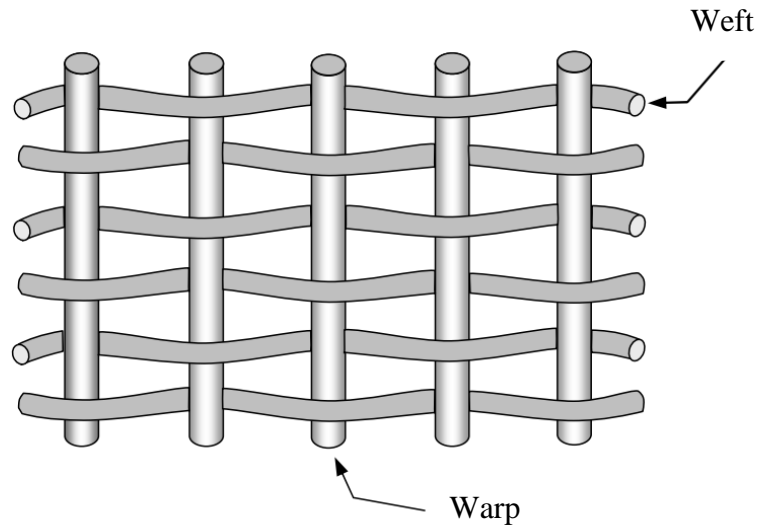


Figure 2.6: Warp and weft of a fabric.

Fabrics differ in the type of yarns used (single yarns, rovings, etc.), therefore in the linear weight of the yarns, and in the way in which the warp and weft yarns are interwoven (or weaved). Figure 2.7 illustrates the classic weaves used: canvas or taffeta, twill, satin, cross weave, unidirectional weave.

a. Taffeta

In the taffeta weave (Figure 2.7), the warp and weft yarns intersect alternately, leading to a very flat, stable, but not very deformable fabric. This weave gives the fabric substantially identical mechanical properties in both the warp and weft directions (for identical weaving yarns). However, the taffeta weave leads to a high degree of fiber curvature which induces a decrease in the mechanical performance of the composites.

b. Twill

In twill weave, the number of warp and weft threads that interlace can vary. In a 2 x 1 twill, the weft threads pass over one warp thread and under two warp threads, and in a 2 x 2 twill, the weft threads pass over two warp threads and under two warp threads. This type of crisscrossing leads to a repeating pattern in the fabric in the form of diagonal ribs. The twill weave has the advantage of being both flexible and dense. The twill fabric thus allows for sliding between the warp and weft threads and adapts well to complex moulding shapes.

c. *Satin*

Satin weave is quite similar to twill, but the number of warp and weft threads which pass over each other before crisscrossing is higher. Each satin fabric is characterized by a number, usually 4 or 8, indicating that the warp threads pass over 4 or 8 weft threads. The result is a fabric with one side containing more warp threads and the other side containing more weft threads. The satin weave is well suited for moulding complex shapes.

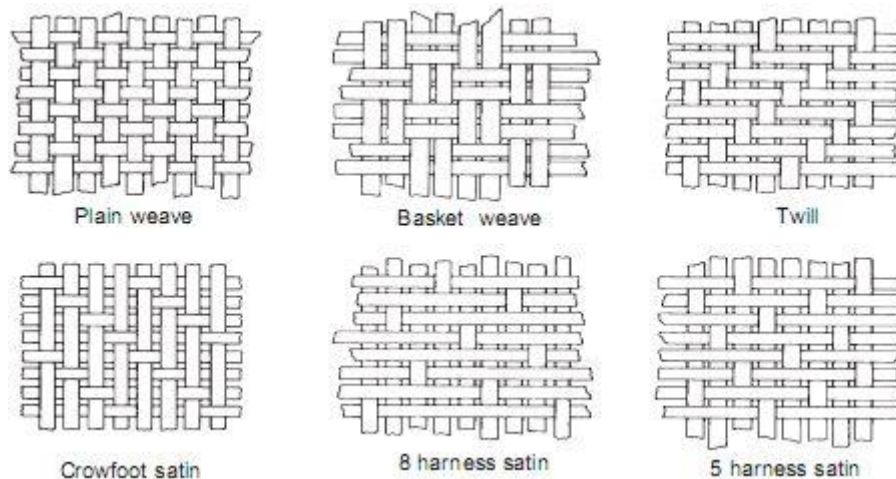


Figure 2.7: The main types of weaves used for weaving fabrics.

d. *Twill fabric (crossed fabric) without crisscrossing*

In a twill fabric, two layers of threads are superimposed without crisscrossing the threads, and joined by a warp and weft of fine threads, having practically no effect on the mechanical performance of the fabric. The absence of crisscrossing eliminates shearing effects and results in a high performance but expensive fabric.

e. *Unidirectional weave fabric*

In a unidirectional fabric, the yarns are aligned parallel to the warp direction, and they are joined by a fine yarn in the weft direction. Thus, the fabric is unidirectional with high performance in the warp direction.

The mechanical performance of the various fabrics depends on:

- ✓ the type of threads constituting the fabric: natural (glass, carbon, etc.), threads with or without twist, etc.

- ✓ Armor. Unidirectional and high modulus weaves give the best performance. Satin weave and, to a lesser extent, twill have superior performance to plain weave.
- ✓ the texture, by the overall reinforcement rate and according to each of the warp and weft directions of the fabric.

2.2.3. Multidirectional woven structures

2.2.3.1. Braids and preforms

It is possible to produce braids or preforms by cylindrical or conical weaving of a tubular fabric. The threads intertwine in a helix, the variation of the pitch (figure 2.8) of which allows the braid to be adjusted to the shape it must cover. It is thus possible to produce a part of revolution with a variable diameter along its arc. By this process, various fabrics can be obtained in the form of conical “socks”, ogival or hemispherical, used to meet the needs of aeronautical construction (re-entry cone, nozzles, etc.).

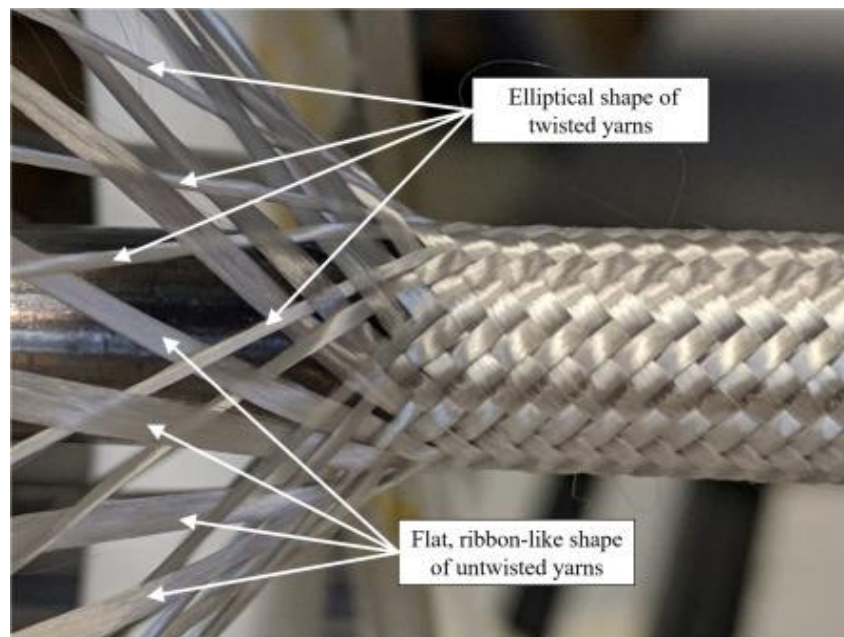


Figure 2.8: Cylindrical weave.

2.2.3.2. Multidirectional fabrics

Volumetric weaves are also used, and characterized by the number of weaving directions: 3D, 4D, etc. The simplest structure is that of 3D weaving, where the threads are arranged in 3

orthogonal directions (figure 2.9). In a 4D weave, the threads are arranged in 4 directions (figure 2.10). The objective is to obtain isotropic composite materials.

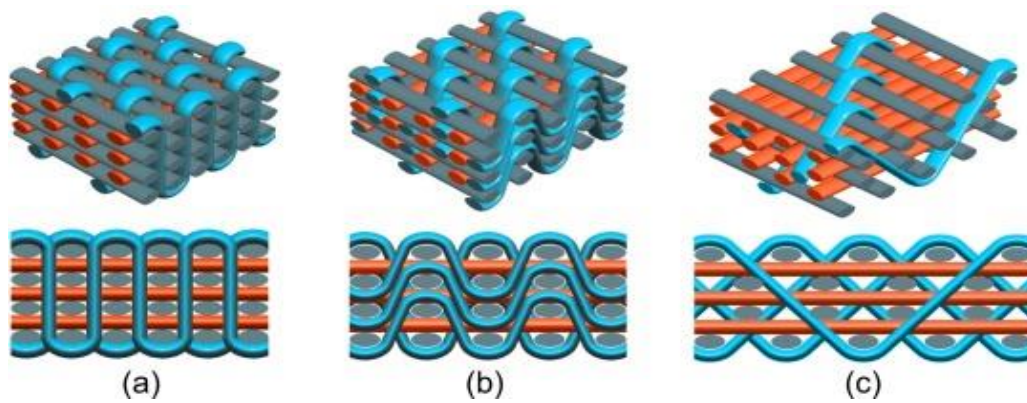


Figure 2.9: Orthogonal 3d weaving

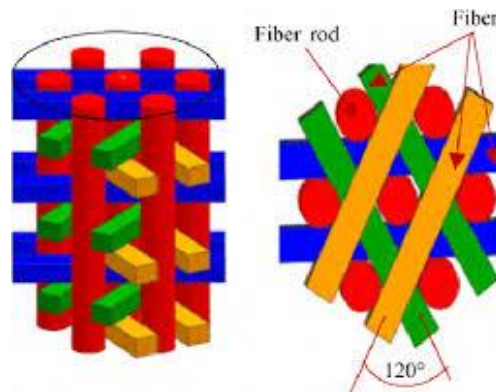


Figure 2.10: 4D weaving.

2.3. Main fibers

2.3.1. Glass fibers



Figure 2.11: Glass fibers.

Glass in solid form is characterized by a very high fragility, attributed to a high sensitivity to cracking. On the other hand, when produced in the form of small diameter fibers (a few tens of micrometres), the glass loses this characteristic and then has good mechanical characteristics. Glass fibers are produced from a spinnable glass, called textile glass, composed of silica, alumina, lime, magnesia, etc. These inexpensive products, combined with fairly simple production processes, give glass fibers an excellent performance/price ratio, which places them by far at the forefront of reinforcements currently used in composite materials.

Depending on their compositions, different types of spinnable glass can be obtained (Table 2.1). In practice, type E glass constitutes almost the entire tonnage of textile glass currently produced. The other glasses, representing a small quantity overall (around 1%), are reserved for specific applications:

- ✓ D glass, with high dielectric properties, for the construction of electronic telecommunications equipment, in particular radomes;
- ✓ C glass, resistant to chemical agents for the surface layers of structures particularly exposed chemically;
- ✓ R and S glasses, with high mechanical characteristics for the production of structures with high mechanical performance.

Table 2.1: Different types of spinnable glasses

Type	General characteristics
E	For general use; good electrical properties
D	High dielectric properties
A	High alkali content
C	Good chemical resistance
R, S	High mechanical resistance

We will hereafter consider only type E and type R glass fibres, the compositions of which are given in Table 2.2. It should be noted that they contain a very low proportion or absence of alkaline oxides, unlike commonly used glasses. This fact leads to high processing temperatures, with technical and economic consequences.

2.3.1.1. Glass fiber production

Glass fibres are produced by drawing molten glass (figure 2.12) through spinnerets, a type of tray made of platinum-rhodium alloy, and pierced at their bases with calibrated orifices of approximately 2 mm in diameter. The molten glass is held in the spinnerets, heated by the Joule effect, at around 1250 °C. At this temperature, the viscosity of the glass allows it to flow by gravity through the orifices, in the form of fibres of a few tenths of a millimetre. At the outlet of the spinneret, the glass in the plastic phase is simultaneously drawn at high speed and cooled. The cooling conditions and drawing speed make it possible to obtain discontinuous fibres of different diameters (generally from 5 to 15 μm).

The monofilaments are then gathered without twisting to form a base thread (called *sillionne* thread), which is wound onto a spool or in the form of a ball. These basic yarns are the most used as glass reinforcements in composite materials.

Discontinuous glass fibers can also be developed and assembled in an assembly called *verranne*. *Veranne* yarns are distinguished from continuous yarns by a fluffy appearance.

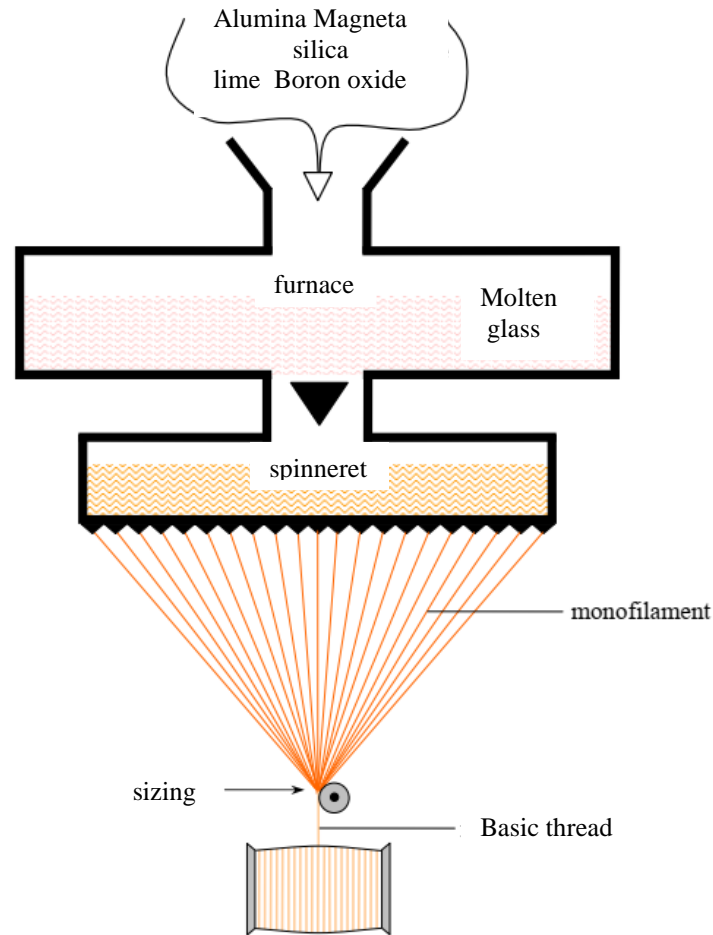


Figure 2.12: Schematic diagram of the mechanical or silionne drawing process.

2.3.1.2. Glass fiber sizing

Glass filaments from the industry cannot be used directly for various reasons:

- ✓ the lack of cohesion between the filaments, which prevents the formation of threads;
- ✓ the sensitivity of the glass to abrasion, leading to deterioration during handling after fiberization;
- ✓ the sensitivity to water attack;
- ✓ the creation of electrostatic charges resulting from various frictions

To overcome these defects, an operation called sizing is carried out at the outlet of the die, which consists of depositing a sizing product of complex composition on the surface of the glass filaments. In light of the defects mentioned above, the various functions of sizing are:

- ✓ establish a certain cohesion between the filaments;
- ✓ give a greater or lesser stiffness to the threads (a thread intended to be cut must be stiff, a thread intended for moulding or weaving must be flexible);
- ✓ protect the filaments against abrasion;
- ✓ avoid the appearance of electrostatic charges;
- ✓ facilitate the impregnation of the filaments by the resin (surface wetting and penetration into the heart of the filaments);
- ✓ promote the glass-resin bond on which the mechanical performance of the composite depends, but also the behaviour of the material during aging, sensitivity to humidity, corrosion, etc.

2.3.1.3. Mechanical properties of glass fibers

It is customary to give as reference mechanical characteristics the characteristics measured on monofilaments taken at the outlet of the die. Table 2.3 gives the usual values of these quantities.

Table 2.3: Mechanical characteristics of type e and r glasses, measured on filaments at the outlet of the die

Characteristics	Glass E	Glass R
Volumetric weight (ρ) kg/m ³	2600	2550
Young's modulus (E_F) GPa	73	86
Breaking stress σ_{fu} MPa	3400	4400
Elongation at break ϵ_{fu} %	4.4	5.2
Poisson's ratio ν_f	0.22	-

Following fiberization, glass filaments are subjected to various mechanical (abrasion, etc.) and chemical (humidity, etc.) stresses which reduce their initial mechanical characteristics. Table 2.4 gives the values of the breaking stress, measured on monofilaments and base yarns that have undergone sizing. The values obtained seem to indicate a drop in the characteristics when the number of filaments increases. In fact, the values measured on the yarns are not really significant because of the difficulties in loading all the filaments constituting the yarn simultaneously and uniformly. In composite materials, the glass-resin bond via the sizing

ensures a fairly homogeneous distribution of the load. The measurements, deduced from the rupture of unidirectional composite materials, lead to the values of stresses and elongation at break of the fibers reported in Table 2.5.

These values, close to those measured on monofilament taken from industrial thread (Table 2.4), must be considered as being representative of the characteristics at break of glass fibers.

Table 2.4: Breaking stresses measured on monofilaments and base yarns (in mpa)

	Glass E	Glass R
Monofilament taken at the outlet of the die	3400	4400
Monofilament taken from industrial silicon thread	2000-2400	3600
Industrial silicone thread with a large number of filaments	1200-1550	1700-2000

Table 2.5: Breaking characteristics of an industrial silionne thread, deduced from the characteristics measured on a unidirectional epoxy resin/glass thread composite.

	Glass E	Glass R
Breaking stress σ_{fu} MPa	2400-2600	3000-3600
Elongation at break ϵ_{fu} %	3.4	4

Finally, it is interesting to note that glass fibers retain their mechanical characteristics up to fairly high temperatures, of the order of 200 °C for E glass and 250 °C for R glass. These fibers are therefore well suited for the reinforcement of resins with high thermal resistance.

2.3.1.4. Industrial textile glass products

a) *Basic Threads*

We have seen that the basic yarns were produced by parallel assembly without twisting of the monofilaments at the outlet of the die, to end up either with continuous yarns (*Silionne* yarns) or with discontinuous yarns (*Veranne* yarns). These yarns are only an intermediary in the production of industrial textile glass products, and are not of direct interest to the manufacturer of composite materials. The basic yarns are characterized by:

- ✓ the designation of the glass used (E, R, etc.),
- ✓ the type of yarn: sillionne (C: continuous) or verranne (D: discontinuous),
- ✓ the nominal diameter of the filaments,
- ✓ the linear weight of the yarn expressed in tex.

For example, we will have the basic thread EC 9 34: E-glass sillionne thread, filament diameter: 9 μm , linear weight of the thread: 34 tex. This designation complies with the ISO 2078 standard.

b) Shredded fibers

These fibers are obtained by grinding the base yarns. Their length is of the order of a few tenths of a millimetre, with a length to diameter ratio of approximately 10 to 40. The ground fibers are used for the reinforcement of certain thermoplastic resins, casting resins, mastics, etc.

c) Basic chopped threads

Chopped base threads are obtained by cutting the sillionne base threads. The most common lengths are: 3, 4.5, 5, 13, 25 and 50 mm. Chopped base threads are used for the reinforcement of various materials, especially thermoplastic resins.

d) Roving

Roving is obtained by assembling sillionne base yarns without twisting, and is presented in balls (without tubes) or in reels (on tubes) (figure 2.13).

The designation of the roving indicates either the linear weight or the number of base yarns (ISO 2078 standard). For example, for EC 10 2400 roving (overall designation), or EC 10 40X60 roving (complete designation), EC 10 40 designates the base yarn, X60 designates the number of base yarns assembled without twisting and 2400 indicates the overall linear weight in tex.

The various types of roving vary according to:

- ✓ the number of base yarns: 2, 8, 15, 30, 60 for example;
- ✓ the overall linear weight: 600, 1200, 2400, 4800, 9600 tex.



Figure 2.13: Different presentations of rovings

There are many uses for roving:

- ✓ cutting for the manufacture of mats, for spray moulding;
- ✓ weaving for the manufacture of fabrics;
- ✓ continuous impregnation: filament winding, pultrusion.

At the textile glass supplier, roving is a step in the manufacture of roving fabrics and cut-strand mats. Excluding this share, the rest of the roving market (moulding, impregnation, etc.) represents more than 40% of the entire textile glass market, all products combined.

e) Curly roving (looped roving)

The curly roving (figure 2.14) is made by assembling “curly” sillionne threads. Due to its structure, looped roving provides reinforcement possibilities in the transverse (or adjacent) direction of the thread axis. Looped roving is mainly used in the weaving of heavy fabrics. This fabric improves the strength of laminates in interlaminar shear between layers of fabric.



Figure 2.14: Curly roving

f) Cut-thread mats

The cut thread mats (figure 2.15) differ from each other by:

- ✓ characteristics of the base thread: type of glass, nominal diameter, linear weight, sizing;
- ✓ the characteristics of the binding, sometimes mechanical (needling), but generally chemical. The binder content then varies from about 3 to 15%, and gives the mat greater or lesser flexibility.

The cutting length of the base yarn is generally 50 mm. The surface weight is commonly 300, 450 and 600 g/m². For specific needs, it is possible to obtain products with surface weightes of less than 300 g/m², or greater than 600 g/m², for example 900, 1200 g/m².



Figure 2.15: Cut-thread mats

g) Continuous-thread mats

Continuous yarn mats (figure 2.16) are characterised by better deformability than chopped yarn mats. Generally, continuous yarn mats are made from base yarns with a low linear density, of the order of 25 tex, and a nominal diameter greater than that of the yarns used for chopped yarn mats.



Figure 2.16: Continuous-thread mats

h) Fabrics

We have presented the various types of fabrics in the general case. Textile glass fabrics are presented in rolls:

- ✓ either of great width, close to a meter for “silionne” or “verranne” fabrics or up to about 3 m in the case of roving fabrics;
- ✓ or of reduced width in the form of ribbons.

The surface weights depend on the yarn reference and the weaving parameters. They commonly range from:

- ✓ 50 to 500 g/m² for silionne or verranne fabrics,
- ✓ 150 to 1,000 g/m² for roving fabrics.

There are also mat-fabric complexes developed to meet specific requirements. Their main application is the reinforcement of parts with flat shapes or large radii of curvature, and more

particularly large dimensions. In such parts subjected to bending stress, the fabric is placed in the area subjected to tension, and the mat in the area subjected to compression, given their respective good suitability for these two types of stresses.

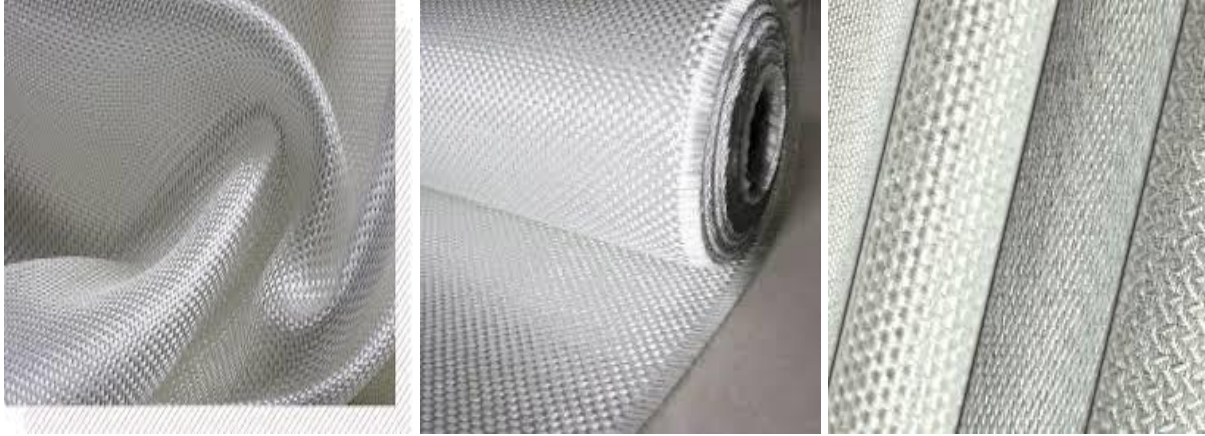


Figure 2.17: Fabrics

2.3.2. Carbon fibers

Graphite has a hexagonal structure of carbon atoms, arranged in parallel crystallographic planes (Figure 2.18). These planes are offset so that a carbon atom projects into the middle of a hexagon of neighbouring planes. The bonds between carbon atoms of neighbouring planes are weak, and give graphite good thermal and electrical conduction properties. On the other hand, the bonds between neighbouring atoms of the same plane are strong, and give graphite high mechanical properties in the direction parallel to the crystallographic planes. The theoretical study of the bonds predicts in this direction a Young's modulus of 1200 GPa and a breaking strength of 20000 MPa.

On the other hand, the low density (less than 2000 kg/m³) leads to remarkably high theoretical specific mechanical properties. These facts explain the numerous developments of different production processes, allowing the most perfect carbon fibers possible to be obtained, and whose crystallographic planes are as parallel as possible to the fiber axis. However, industrial fibers do not reach the theoretical mechanical values, due to the imperfections of the crystalline structures obtained. The characteristics of the fibers produced remain high, however, and can now reach, for the most efficient fibers, around 650 GPa for the Young's modulus and 4000 MPa for the breaking stress.

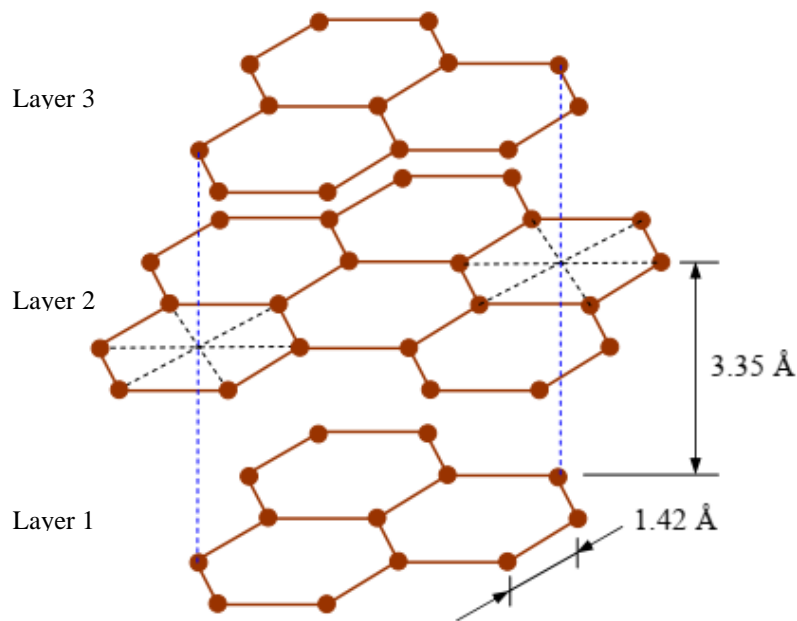


Figure 2.18: Graphite crystal structure.

2.3.2.1. Production of carbon fibers

2.3.2.1.1. From acrylic fibers

Carbon fibers are made from a base polymer called a precursor, which itself comes in the form of oriented and crosslinked fibers. Currently, the fibers used are acrylic fibers made from polyacrylonitrile (PAN). These acrylic fibers are known under various trade names: crylor, courtelle, dralon, orlon, etc. The quality of the final carbon fibers depends on the qualities of the precursor. The principle of development is to subject the acrylic fibers to thermal decomposition, without fusion of the fibers, resulting in graphitization in which the initial structure of the fibers is found. Current processes use strands of acrylic filaments assembled without twisting (generally 500, 1000, 6000, 10000, etc. filaments), and subject them to four successive treatments: oxidation, carbonization, graphitization and surface treatment (figure 2.19).

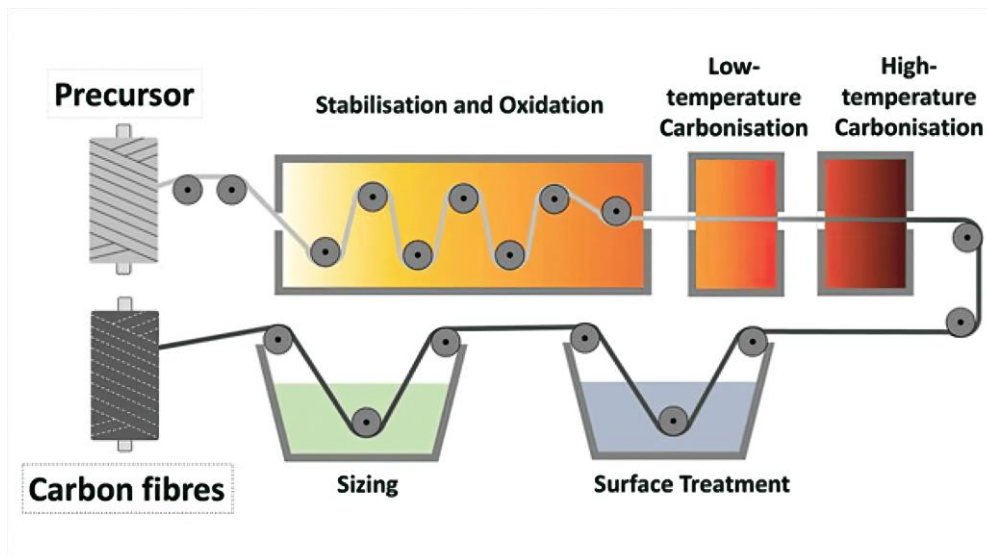
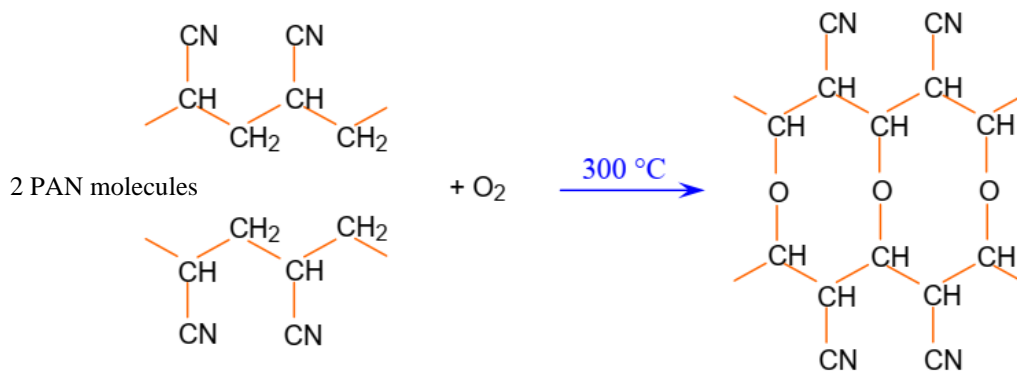


Figure 2.19: Manufacturing process of carbon fibers.

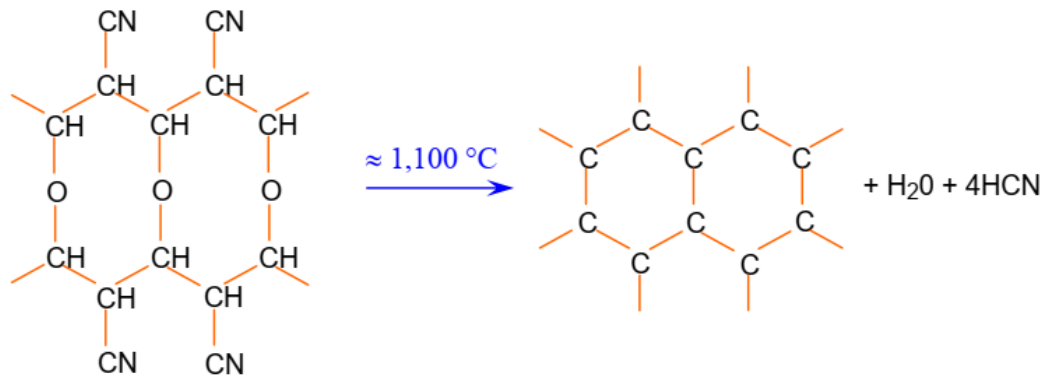
a) Oxidation

Since acrylic fibres are meltable, the purpose of the oxidation phase is to artificially suppress the melting point. This operation is carried out by heating the fibres to around 300°C in an oxygen atmosphere. Oxidation then occurs, leading to cross-linking of the molecular chains and the creation of a three-dimensional network:



b) Carbonization

The second phase consists of gradually heating the crosslinked fibers from 300 °C to approximately 1100 °C, in an inert atmosphere. The water and hydrocyanic acid are then eliminated, with only the carbons being retained in the chain:



The fibers obtained after this phase have good mechanical characteristics and can be used after surface treatment (figure 2.19). The fibers are then called HR fibers (high resistance) or HT fibers (high tenacity).

c) *Graphitization*

The graphitization phase is used when fibers with a high Young's modulus are desired. This phase consists of carrying out, following carbonization, a pyrolysis of the fibers, in an inert atmosphere, up to 2600 °C or at higher temperatures. Graphitization causes a reorientation of the hexagonal carbon networks along the fiber axis, which results in an increase in the Young's modulus. However, simultaneously with this reorientation, defects are created in the structure, leading to a decrease in the breaking stress. Depending on the graphitization rate, HM fibers (high modulus fibers) or THM fibers (very high modulus) are obtained.

d) *Surface treatment*

The last phase of production consists of a surface treatment, by controlled oxidation in an acidic environment (nitric or sulfuric). The purpose of this phase is to increase the roughness of the filaments as well as the nature of the chemical bonds, in order to improve the fiber-resin bond.

2.3.2.1.2. From the precursor pitch

Since the 1970s, carbon fiber production processes have been developed from pitch, which is a refinery residue from oil or coal. In this process, the pitch is heated to 350 °C- 450 °C to obtain a mesophase (intermediate between liquid and crystal), then spun to improve

orientation. As in the PAN process, the filaments are oxidized and carbonized, and finally pyrolyzed at temperatures above 2000 °C to obtain high modulus fibers.

The carbon fibers produced by this process have various advantages:

- ✓ a high filament/precursor weight yield of around 75 to 90% (50% for the PAN process),
- ✓ a higher graphitization speed,
- ✓ a cheap raw material.

The development of this technique should allow carbon fibers to reach large industrial markets (such as the automobile industry, etc.), by significantly reducing the cost price, compared to fibers obtained using the PAN precursor.

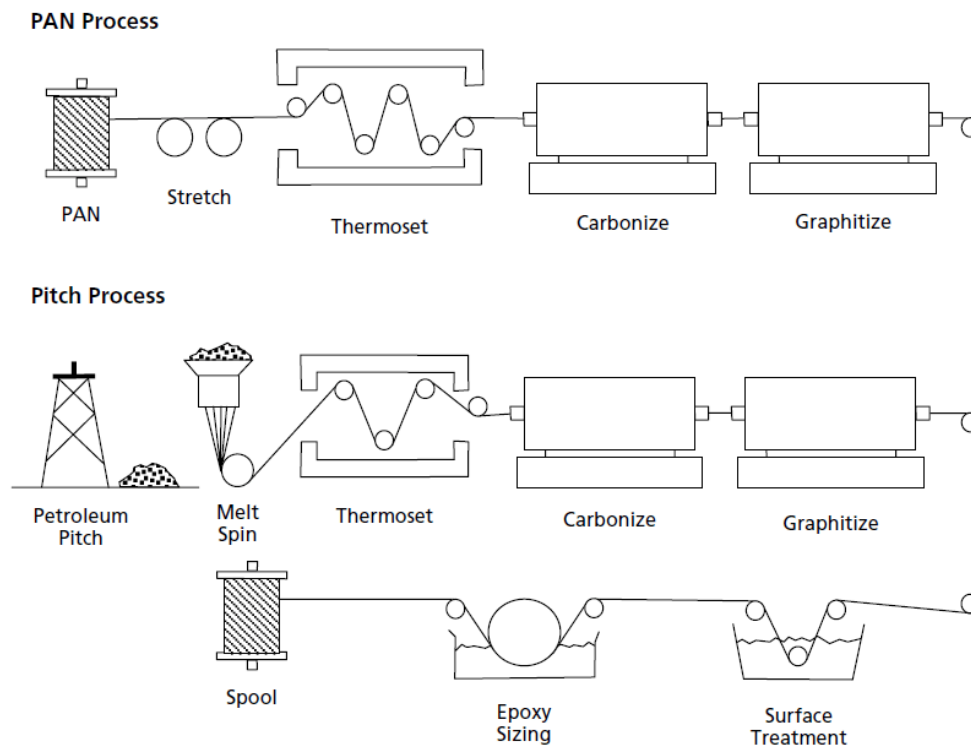


Figure 2.20: PAN and Pitch carbon fibers manufacturing process.

2.3.2.2. Mechanical characteristics of carbon fibers

Carbon fibres have very good mechanical characteristics, especially since their density is low (generally less than 2000 kg/m³). Table 2.6 compares the characteristics of carbon fibres with those of E-glass fibres. In addition, it should be noted that carbon fibres have excellent

temperature resistance in a non-oxidising atmosphere. Indeed, their mechanical characteristics are maintained up to around 1500 °C. This property has led to the development of carbon fibre/carbon matrix composites with high thermal resistance, used in rocket nozzles, brake pads (trucks, Formula 1, aircraft), furnace elements, etc. These materials, coated with an anti-oxidising protective layer, also find applications in oxidising atmospheres in the space sector: leading edges, tiles, etc.

Table 2.6: Characteristics of carbon fibers, compared to those of e-glass fibers

Characteristics	Glass E	Carbon HR	Carbon HM	Carbon THM	Carbon HM (Pitch)
Volumetric weight (ρ) kg/m ³	2600	1750	1810	1950	2000
Diameter (μm)	10 to 20	5 to 7	5 to 7	5 to 7	12
Young's modulus (E_F) GPa	73	230	400	600	280
Specific modulus (E_F / ρ) MNm/kg	28	130	210	310	40
Breaking stress σ_{fu} MPa	3400	3000-4000	2800	2000	2000-2400
Specific stress (σ_{fu} / ρ) kNm/kg	1300	1710-2290	1550	1030	1000-1200
Cost price reported to E glass fibers	1*	10-15	30-50	200-400	50-100

1* \approx 4€/kg in 2009 HM (pitch): fibres produced from pitch.

2.3.2.3. Industrial products

The basic product is the continuous yarn or carbon roving, made of carbon monofilaments (for example: 500, 1000, 3000, 6000, 10000, etc. filaments) assembled without twisting. These rovings can be used directly for composite manufacturing processes by pultrusion or filament winding. The rovings can also be woven to obtain ribbons, braids, unidirectional or multidirectional fabrics, hybrid fabrics (glass-carbon, Kevlar-carbon), etc.

These various forms can optionally be pre-impregnated.

The rovings can also be cut to lengths of a few millimetres and used for the production of filled resins.

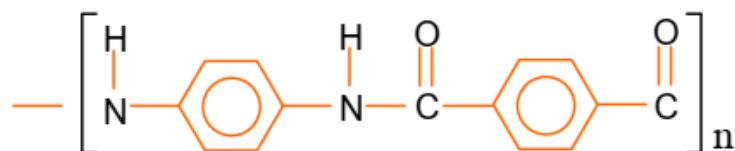
2.3.3. Aramid fibers with high mechanical characteristics



Figure 2.21: Aramide fibers.

Aramid fibers with high mechanical properties are generally known under the name of “Kevlar”, the trade name of the fiber developed by Dupont de Nemours (USA), and marketed in 1972. Other groups also develop aramid fibers, in particular the AKZO group (Germany-Holland) which produces aramid fibers under the name of “Twaron” fibers, and the Japanese group Teijin Ltd “Technora” fibers.

Aramid fibers are synthetic polyaramid or aromatic polyamide fibers whose structure is made up of benzene nuclei linked by CO and HN groups:



Aramid fibers are produced by low-temperature synthesis (-10 °C), followed by solution spinning. The fibers are then stretched and heat treated to increase their modulus of elasticity.

2.3.3.1. Characteristics

The mechanical characteristics of aramid fibres are reported in Table 2.7 for monofilaments. On multifilament threads, the characteristics are generally lower. The table shows a high specific breaking stress, of the same order of magnitude as high-resistance carbon fibres HR (Table 2.6), however with a cost price 4 to 6 times lower. Nevertheless, their use is limited by various weaknesses of aramid fibre composite materials:

- ✓ low compressive, bending and buckling strengths;
- ✓ sensitivity to interlaminar shear.

These weaknesses are generally attributed to poor fibre-matrix adhesion. To remedy this, suitable sizing is developed. Hybrid composites (glass-Kevlar, carbon-Kevlar) are also used.

Table 2.7: Mechanical characteristics of aramid monofilaments

Characteristics	Kevlar 29	Kevlar 49	Kevlar 149	Twaron	Technora
Volumetric weight (ρ) kg/m ³	1440	1450	1470	1440	1390
Diameter (μm)	12	12	12	12	12
Young's modulus (E_F) GPa	60	120	160	60	90
Specific modulus (E_F / ρ) MNm/kg	42	83	110	42	65
Breaking stress σ_{fu} MPa	3000	3000	2400	2600	2800
Specific stress (σ_{fu} / ρ) kNm/kg	2080	2070	1630	1800	2010
Elongation at break (%)	3.6	1.9	1.5	3	4

2.3.3.2. Industrial uses

Aramid fibers are available as rovings, fabrics, braids, and ribbons. One of their first mass-market uses was to replace steel reinforcement in tires, belts and hoses. Given their lightness and their good resistance to shock, impact and abrasion, aramid fibers are used for the manufacture of certain sporting articles (skis, tennis rackets, etc.) and for the manufacture protection against projectiles (armouring, bulletproof vests, helmets, gloves, etc.), friction products and joints, cable reinforcement, etc.

2.3.4. Ceramic fibers



Figure 2.22: Ceramic fibers.

Various fibers of refractory or ceramic materials (carbides, borides, nitrides, etc.) can be produced by chemical vapor deposition on a support thread. Currently, the fibers obtained by this process, and which are the subject of production, are:

- ✓ boron (B) fibers,
- ✓ boron (B)-boron carbide (B_4C) fibers,
- ✓ silicon carbide (SiC) fibers,
- ✓ boron-silicon carbide fibers, called BorSiC.

The fibres are in fact large filaments (diameter of the order of $100\ \mu\text{m}$), formed of a core (the support thread) made of tungsten or carbon with a diameter of approximately $10\ \mu\text{m}$, covered with:

- ✓ a layer of boron approximately $40\ \mu\text{m}$ thick (boron fibres),
- ✓ a layer of boron $40\ \mu\text{m}$ thick and a layer of boron carbide $4\ \mu\text{m}$ thick (B- B_4C fibres),
- ✓ a layer of silicon carbide (SiC fibres),
- ✓ a layer of boron and a layer of silicon carbide (BorSiC fibres).

2.3.4.1. Mechanical characteristics and use

The mechanical characteristics of the various fibres are quite similar (e.g. Table 2.8). These characteristics are maintained up to temperatures ranging from 500 to $1000\ ^\circ\text{C}$.

Due to a high-cost price, the use of these fibres is limited. Currently, SiC and BorSiC fibres are mainly used with metal matrices (e.g. aluminium) or ceramic matrices, leading to very high-cost composite materials, used for compressor blades, turbines, etc.

Table 2.8: Mechanical characteristics of boron, boron-boron carbide and silicon carbide fibers.

Characteristics	Boron	Boron + B ₄ C	SiC
Volumetric weight (ρ) kg/m ³	2600	2600	3000
Diameter (μm)	100-150	100-150	100-150
Young's modulus (E_F) GPa	430	430	410
Specific modulus (E_F / ρ) MNm/kg	165	165	140
Breaking stress σ_{fu} MPa	3800	4000	3900
Specific stress (σ_{fu} / ρ) kNm/kg	1460	1540	1300

Boron fibers and boron-silicon carbide fibers are available in the form:

- ✓ continuous yarns made of parallel monofilaments,
- ✓ pre-impregnated ribbons for filament winding,
- ✓ unidirectional fabrics.

These fibres combined with epoxy resins, with which adhesion is very good, or with other thermoplastic or thermosetting resins, are used in the fields of sports and leisure.

2.3.4.2. Other generations of ceramic fibers

Other generations of ceramic fibers made of silicon carbide (SiC) or silicon carbotitanate (SiCTi) are obtained by the precursor route, following a process comparable to that of the development of carbon fibers via polyacrylonitrile. This precursor route leads, unlike the previous process, to ceramic fibers of small diameters. The mechanical characteristics of these fibers are reported in Table 2.9. These fibers have good stability of their properties up to temperatures of 1200 °C to 1600 °C.

Ceramic fibers based on alumina are also developed, such as alumina fibers (Al₂O₃), aluminosilicate fibers (Al₂O₃, SiO₂), Boro-aluminosilicate fibers (Al₂O₃, SiO₂, B₂O₃). The

mechanical characteristics of these fibers, obtained by soil-gel process, are reported in Table 2.10.

Table 2.9: Mechanical characteristics of SiC, SiNC and SiCTi fibers.

Characteristics	SiC Nicalon	SiNC HPZ	SiCTi Tyranno
Volumetric weight (ρ) kg/m ³	2550	2350	2400
Young's modulus (E_F) GPa	180-200	170	200
Specific modulus (E_F / ρ) MNm/kg	70-80	70	85
Breaking stress σ_{fu} MPa	3000	2400	3000
Specific stress (σ_{fu} / ρ) kNm/kg	1200	1000	1250

Table 2.10: Mechanical characteristics of alumina-based ceramic fibers.

Characteristics	Alumina Al ₂ O ₃	Alumino-silicate Al ₂ O ₃ , SiO ₂	Borosilicoaluminate Al ₂ O ₃ , SiO ₂ , B ₂ O ₃
Volumetric weight (ρ) kg/m ³	3400-3950	3100-3200	2700-3100
Young's modulus (E_F) GPa	300-390	190-250	150-200
Specific modulus (E_F / ρ) MNm/kg	90-100	60-80	55-65
Breaking stress σ_{fu} MPa	1500-2000	2100-2200	1700-1800
Specific stress (σ_{fu} / ρ) kNm/kg	440-500	685	580-630

2.3.5. Thermostable synthetic fibers

Thermostable synthetic fibers are organic fibers obtained by synthesis, and which retain their mechanical characteristics at high temperatures. Combined with thermostable resins, they make it possible to obtain materials whose mechanical characteristics at temperature are retained.

The mechanical characteristics of these fibers are however significantly lower than those of usual fibers. They are used in electrical and thermal insulation, thermal protection: missile shields, space vehicle re-entry cones, etc.

2.3.6. Other fibers

There are various other fibers used for specific applications. These fibers are generally low modulus and breaking stress, except for metallic fibers. Generally, their use is reserved for research into:

- ✓ low-cost products,
- ✓ high thermal insulation products,
- ✓ products with good thermal or electrical conductivity,
- ✓ high sound absorption products,
- ✓ etc.

Among these fibers we have:

1. Fibers of plant origin, such as wood used in the form of oriented fibers, sisal, jute, linen, etc.



Figure 2.23: Natural (plant) fibers.

2. Synthetic fibers, such as polyester fibers (tergal, dacron, terylene, etc.), polyamide fibers, polyethylene fibers, polypropylene fibers, etc.

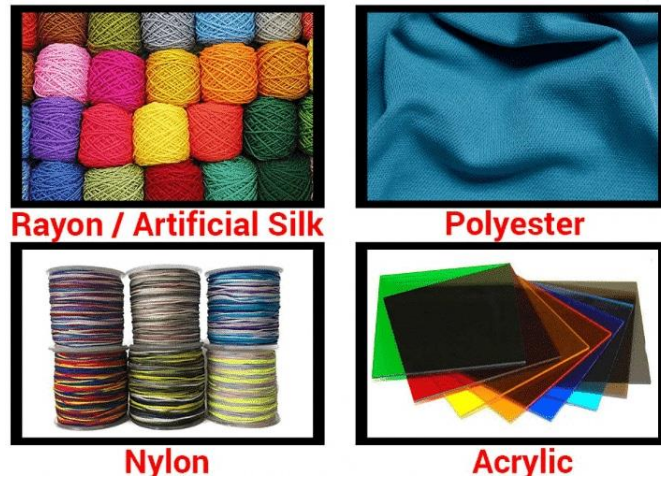


Figure 2.24: Synthetic fibers.

3. Metallic fibers, such as steel, copper, aluminium fibers. These fibers are used with metal matrices for their good thermal and electrical conductivity and their high thermomechanical characteristics.



Figure 2.25: Metallic fibers.

2.4. Resins (matrices)

The role of resins used in composite materials is to transfer mechanical stresses to the fibres and protect them from the external environment. The resins must therefore be sufficiently deformable and have good compatibility with the fibres. In addition, they must have a low density in order to maintain high specific mechanical characteristics of the composite materials.

Given these constraints, the resins used are polymers, modified by different adjuvants and additives: release agents, stabilizers, pigments, etc. The resins are delivered in solution, in the form of non-crosslinked polymers suspended in solvents that prevent bridging between the prepolymerized macromolecules. Under the action of heat, bonds develop between the prepolymer chains to form a crosslinked polymer following a three-dimensional structure. There are two main families of polymer resins: thermoplastic resins and thermosetting resins. These two types of resin have the ability to be moulded or shaped, to give either a finished product or a semi-finished product whose shape can be modified.

Thermoplastic resins, whose manufacturing reaches by far the largest tonnage due to their low cost, have the property of being able to be shaped several times by successive heating and cooling. These resins can therefore be recovered and easily recycled.

On the other hand, thermosetting resins can only be shaped once. Indeed, after polymerization by heat input in the presence of a catalyst, these resins lead to a geometric structure that can only be destroyed by a significant input of thermal energy. Thus, thermosetting resins have higher mechanical and especially thermomechanical properties than thermoplastic resins.

Due to these higher characteristics, thermosetting resins are currently the most widely used in the implementation of composite materials. However, the improvement of the characteristics of thermoplastic resins leads to a use that continues to grow.

Two other classes of resins for specific uses are also used, these are:

- ✓ thermoplastic resins which can withstand, in continuous service, temperatures of around 200°C and above,
- ✓ elastomers whose reinforcement by different fibers leads to various applications in the automotive sector.

2.4.1. Thermosetting resins

The main thermosetting resins used in the implementation of composite materials are, in decreasing order of tonnage:

- ✓ unsaturated polyester resins: condensed polyesters, vinyl esters, allylic derivatives, etc.,
- ✓ Condensation resins: phenolic, aminoplast, furanic, etc.,
- ✓ epoxy resins.

2.4.1.1. Polyester resins

Unsaturated polyester resins come by far in the lead when it comes to the use of composite materials. Their development is the result of:

- ✓ low production costs,
- ✓ their diversity offering multiple possibilities,
- ✓ adaptation to manufacturing processes that are easy to implement and automate.

Hence an ever-increasing industrial development.

Depending on their modulus of elasticity, polyester resins are classified into: flexible resins, semi-rigid resins and rigid resins. The resins usually used in the implementation of composite materials are of the rigid type, and we will retain the following characteristics for these hardened resins:

Volumetric weight	1200kg/m ³
Modulus of elasticity in traction	2.8-3.5 GPa
Modulus of elasticity in bending	3-4.5 GPa
Tensile break stress	50-80 MPa
Bending break stress	90-130 MPa
Tensile elongation at break	2-5%
Elongation at break in flexion	7-9%
Compressive resistance	90-200 MPa
Shear strength	10-20 MPa
Bending temperature under load (1.8 MPa)	60-100°C

Among the advantages of unsaturated polyesters, we will note:

- ✓ good rigidity resulting from a fairly high modulus of elasticity,
- ✓ good dimensional stability,

- ✓ good wettability of fibers and fabrics,
- ✓ ease of implementation,
- ✓ good chemical resistance,
- ✓ low production cost,
- ✓ good chemical resistance to hydrocarbons (gasoline, fuel, etc.) at room temperature, etc.

Among the disadvantages, we will note:

- ✓ poor temperature resistance: less than 120 °C in continuous service,
- ✓ sensitivity to cracking, mainly in the case of impacts,
- ✓ significant shrinkage of the order of 8 to 10%,
- ✓ poor behaviour in steam, boiling water with risk of hydrolysis, hence the need to cover polyester resin composite materials with a layer of “gel-coat” in order to make them waterproof,
- ✓ degradation in light by ultraviolet rays,
- ✓ flammability.

2.4.1.2. Condensation resins

Condensation resins include phenolic resins, aminoplasts and furan resins.

1. **Phenolic resins** are the oldest of the thermosetting resins, the best known of which is Bakelite. The characteristics of these resins are as follows:

Volumetric weight	1200 kg/m ³
Modulus of elasticity in bending	3 GPa
Tensile break stress	40 Mpa
Tensile elongation at break	2.5 %
Bending break stress	90 Mpa
Compressive resistance	250 Mpa
Bending temperature under load	120°C

Among the advantages, we will mention:

- ✓ excellent dimensional stability,
- ✓ good resistance to heat and creep,

- ✓ good resistance to chemical agents,
- ✓ low shrinkage,
- ✓ good mechanical characteristics,
- ✓ low cost.

Among the disadvantages, we will note:

- ✓ implementation under pressure, therefore at low rates,
- ✓ the dark colours of the resins,
- ✓ non-suitability for food uses.

Phenolic resins will therefore be used in the case of parts requiring high temperature resistance or good resistance to chemical agents.

2. The characteristics of *aminoplast resins* are similar to those of phenolic resins. To the advantages of these resins, we must add:

- ✓ the possibility of food uses,
- ✓ the possibility of colouring the resins.

3. *Furan resins* are rarely used in France because of their cost, three times higher than polyester resins. Their advantages include:

- ✓ faster hardening than phenolic resins,
- ✓ high inertia with respect to corrosive chemical agents.

This last characteristic leads to the use of furan resins in the case of materials that must resist chemicals: tanks, pipes, containers, etc.

2.4.1.3. Epoxy resins

The most widely used resins after unsaturated polyester resins are epoxy resins. However, they only represent around 5% of the composite market, due to their high price (around five times more than that of polyester resins).

Due to their good mechanical characteristics, epoxy resins, generally used without fillers, are the matrices of high-performance composites (aeronautical construction, space, missiles, etc.).

The general mechanical characteristics of epoxy resins are as follows:

Volumetric weight	1100 to 1500 kg/m ³
Modulus of elasticity in tensile	3-5 GPa
Tensile break stress	60-80 MPa
Bending break stress	100-150 MPa
elongation at break	2-5 %
shear resistance	30-50 MPa
Bending temperature under load	290°C

Epoxy resins therefore lead to a set of high performances. However, to really benefit from these performances, it is necessary to have very long transformation and especially annealing times (from several hours to several tens of hours), at relatively high temperatures (50 to 100 °C).

Among the advantages of epoxy resins, we can highlight:

- ✓ good mechanical properties (in traction, bending, compression, impact, creep, etc.) superior to those of polyesters,
- ✓ good performance at high temperatures: up to 150°C to 190°C continuously;
- ✓ excellent chemical resistance,
- ✓ low moulding shrinkage (0.5 to 1%),
- ✓ very good wettability of reinforcements,
- ✓ excellent adhesion to metallic materials.

Among the disadvantages, we can mention:

- ✓ a long polymerization time,
- ✓ a high cost,
- ✓ the need to take precautions during implementation,
- ✓ sensitivity to cracking.

2.4.2. Thermoplastic resins

The family of thermoplastic resins (we speak of “plastics”) is very vast, and can be separated into mass-produced plastics and technical plastics (or techno-polymers).

Mass-produced plastics are implemented either by injection to obtain moulded objects, or by extrusion to obtain films, plates, tubes, profiles, etc. Technical plastics are generally implemented by injection.

Among the thermoplastic resins, we can mention: polyvinyl chloride (PVC), polyethylene, polypropylene, polystyrene, polyamide, polycarbonate, etc. The interest of thermoplastics lies in their low cost, resulting from both available raw materials and manufacturing processes (injection, extrusion). However, this low cost is linked to weak mechanical and thermomechanical properties. We give below some characteristics for polypropylene and polyamide.

Table 2.11: Some characteristics for polypropylene and polyamide

Characteristic	Polypropylene	Polyamide
Volumetric weight (kg/m ³)	900	1140
Break stress (MPa)	20-35	60-85
Modulus of elasticity (Young's modulus) (GPa)	1.1-1.4	1.2-2.5
Bending temperature under load (°C)	50-60	65-100

Various thermoplastics can be reinforced with fibres and are then part of composite materials. However, in the field of composites, thermoplastic resins have limited development, due to the need to use high-temperature transformations of solid products.

2.4.3. Thermostable resins

Thermostable resins are distinguished from other resins, previously considered, essentially by their thermal performances which retain their mechanical properties for temperatures higher than 200°C. In practice we find for these resins the two main families of thermoplastic and thermosetting resins.

Thermostable resins are developed especially in the fields of aviation and space, where laboratories seek to develop new resins. Among thermostable resins, bismaleimide and polyimide resins are the most used.

Bismaleimide resins are resins whose network is developed at temperatures of 180 to 200°C. The moulding processes are identical to those of composites with a thermosetting matrix of the polyester or epoxy type.

Polyimide resins appeared on the market around 1970. They are resins with high thermal resistance, but very high price. These resins make it possible to obtain composites with a resistance greater than that of aluminium at 250°C.

2.5. Fillers and additives

Different products can be incorporated into the resin to give it special characteristics or reduce its cost. The quantity of the products added can vary from:

- ✓ a few tens of % in the case of fillers,
- ✓ to a few % or less in the case of additives.

The addition of these products has the function either of improving the mechanical and physical characteristics of the finished product, or of facilitating its implementation. We give examples of fillers and additives in this paragraph.

2.5.1. Fillers

2.5.1.1. Strengthening (reinforcing) fillers

The purpose of incorporating reinforcing fillers is to improve the mechanical characteristics of the resin. These fillers can be classified according to their geometric shape into:

- ✓ spherical fillers
- ✓ non-spherical fillers.

2.5.1.1.1. Spherical fillers

The main interest of these fillers lies in their spherical shape which avoids stress concentrations and, consequently, reduces the susceptibility to matrix cracking compared to non-spherical fillers. Spherical fillers are in the form of spheres, generally called microbeads or microspheres. These solid or hollow spheres have a diameter generally between 10 and 150 μm . They can be made of glass, carbon or organic material (epoxy, phenolic, polystyrene, etc.). Hollow glass microbeads represent more than 99% of the spherical fillers used.

a) Hollow glass microbeads

The main advantage of hollow glass microbeads is a low density (100 to 400 kg/m^3), providing an increase in the specific modulus of the loaded resin and its compressive strength.

Hollow glass microbeads are manufactured by passing fine glass particles containing an expansion gas (usually a mixture of nitrogen and carbon dioxide) through a high-temperature zone. When the temperature of the particles rises, the gas is expanded in the molten glass particle. The particles are then cooled rapidly, causing the microbead wall to solidify before the gas pressure decreases. The microbeads thus obtained have a diameter of around 20 to 130 μm , with a wall thickness of 0.5 to 2 μm . The microbeads can then be selected according to their dimensions. Finally, the microbeads are subjected to surface treatments to improve the bonding of the microbeads with the resin.

Hollow glass microbeads are most often incorporated into epoxy or polyester resins. Their use is limited to low pressure applications due to the low crushing resistance of hollow spheres.

The main advantages of their incorporation into a resin are:

- ✓ reduction of the volumetric weight,
- ✓ increase of the modulus of the resin,
- ✓ improvement of the compressive strength.

b) Other microbeads

There are other hollow microbeads:

- ✓ Carbon microbeads:
 - Volumetric weight 120 kg/m³
 - Diameter 5 to 150 μm
- ✓ Organic microbeads (epoxy, phenolic, etc.):
 - Volumetric weight 100 to 500 kg/m³
 - Diameter 10 to 800 μm

These microbeads are generally more expensive (up to 5 times for carbon microbeads) than glass microbeads.

Other microbeads used include solid glass microbeads. Compared to hollow glass microbeads, the characteristics of solid microbeads are:

- ✓ High volumetric weight: 2,500 kg/m³,
- ✓ Lower price,
- ✓ Possibility of implementation with resins at high pressures

2.5.1.1.2. Non-spherical fillers

Among the non-spherical reinforcing fillers, mica is the most widely used material. It is then incorporated in the form of flakes with dimensions of 100 to 500 μm, and thickness of 1 to 20 μm. Mica is added to thermoplastic or thermosetting resins, for electrical or electronic applications.

2.5.1.2. Non-reinforcing fillers

Non-reinforcing fillers serve either to reduce the cost of resins while maintaining the performance of the resins, or to improve certain properties of the resins.

2.5.1.2.1. Low-cost fillers

These fillers are extracted from rocks or ores, hence their low cost. Generally, the incorporation of these fillers leads to:

An increase of:

- ✓ the volumetric weight of the matrix,
- ✓ the modulus of elasticity,

- ✓ the hardness,
- ✓ the viscosity,
- ✓ the dimensional stability.

A decrease of:

- ✓ the price,
- ✓ the tensile and bending strength.

The main fillers are:

- ✓ carbonates: chalks or calcites (CaCO_3). These are the most commonly used fillers,
- ✓ silicates: talc, kaolin, feldspar, wollastonite,
- ✓ silicas, obtained by grinding and sieving quartz sand.

2.5.1.2.2. Flame retardant (fireproofing) fillers

These fillers added to the resins have the role of reducing or preventing combustion phenomena. Among the solid fillers used in thermosetting resins, we cite:

- ✓ alumina hydrate, the most widely used product in thermosetting resins,
- ✓ antimony oxide.

2.5.1.2.3. Conductive and antistatic fillers

Organic resins are thermal and electrical insulators. For some applications, it is therefore necessary to add a conductive element. The main fillers used are:

- ✓ metal powders or flakes: copper, iron, aluminium, etc.,
- ✓ metallized glass microbeads (copper, silver),
- ✓ carbon particles (carbon black),
- ✓ metal filaments.

2.5.2. Additives

Additives are found in small quantities (a few % or less) and act as:

- ✓ Lubricants and mould release agents,
- ✓ pigments and dyes,
- ✓ anti-shrinkage agents,
- ✓ anti-ultraviolet agents.

2.5.2.1. Lubricants and mould release agents

These additives are intended to facilitate the shaping of the resin and to reduce the tendency of the resin to adhere to moulds, mandrels, etc.

2.5.2.2. Pigments and dyes

Pigments are insoluble products in the form of powders or flakes. They are obtained from metal oxides or salts. From these pigments, it is possible to obtain colouring pastes consisting of pigment dispersions in a paste (resin, plasticizer), for easy use.

Dyes are organic compounds soluble in a suitable solvent. Their use is limited, due to poor chemical and thermal resistance.

2.5.2.3. Anti-shrinkage agents and creep agents

The polymerization of resins leads to a reduction in the interatomic distances of the initial monomer. This results in shrinkage of the polymerized resin, which can result in poor surface finish, warping or micro-cracking of the moulded parts. Although the incorporation of fillers into the resin limits shrinkage, it is often necessary to add specific anti-shrinkage products (so-called “low profile” and “low shrink” additives), which reduce or eliminate the shrinkage phenomenon. These products also improve the flow of the material in certain moulding techniques. These anti-shrinkage agents are generally products based on thermoplastics or elastomers, in powder form or in solution in styrene.

2.5.2.4. Anti-ultraviolet agents

Anti-ultraviolet agents serve to protect resins from the action of ultraviolet rays contained in solar radiation. The principle of these agents is to absorb ultraviolet radiation and thus prevent premature deterioration of the resin by breaking atomic bonds or by passing to an excited state which promotes oxidation (photo-oxidation phenomenon).

2.6. Architecture of composite

Composite material structures can be classified into three types:

- ✓ Single layers
- ✓ Laminates
- ✓ Sandwiches

2.6.1. Single layers

The single-layer composite (elementary layer or stratum) corresponds to the elementary unit of thickness, it is made up of one or more identical layers (woven, UD, mats, etc.) assembled without any orientation. A ply is a semi-finished product of composites (fibers and matrix) presented in a quasi-two-dimensional form, a thin sheet (≈ 0.125 mm). We distinguish between the UD ply (The reinforcement has a UD structure), the woven ply (The reinforcement has an oriented structure) and the mat ply (The reinforcement has a random structure).

The superimposition of the single layers will constitute the laminate (Figure 2.26).

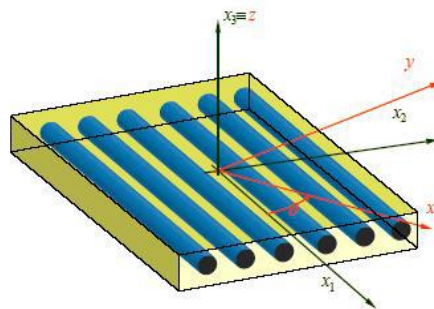


Figure 2.26: Single-layer composite material.

2.6.2. Laminates

Laminates are made up of successive layers (sometimes called plies) of reinforcements (threads, rovings, mats, fabrics, etc.) impregnated with resins (figure 2.27).

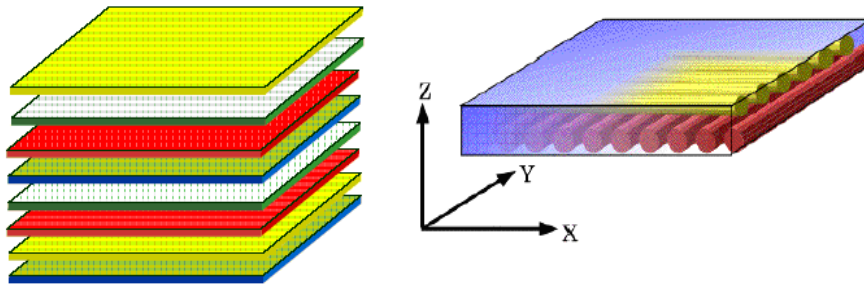


Figure 2.27: Laminate structure.

Moreover, a laminate is made up of a stack of monolayers (figure 2.26) each having its own orientation relative to a reference axis and designated as the laminate reference axis. The choice of the stack and more particularly the orientations will allow to have specific mechanical properties.

Laminates are, with sandwich panels, the composite materials that have the best structural performances. The basic idea is to superimpose anisotropic layers, reinforced with long, uni- or bi-directional fibers, by orienting the layers so as to obtain a final material having the desired properties, in terms of elastic behaviour, rigidity, resistance, etc. The design of the material therefore becomes a phase of the structural design, their qualities are to be:

- ✓ High mechanical performances;
- ✓ Lightweight.

One of the advantages of laminates is the possibility of orienting the fibers in directions adapted to the forces imposed on the structure. The design of a laminated structure therefore involves the choice of materials, fiber orientations and the arrangement of the plies (sequence of stacking plies of given inclination).

2.6.2.1. Unidirectional yarn or fabric laminates

Unidirectional yarn or fabric laminates are a basic type of laminate to which any other type of laminate can theoretically be reduced. These laminates consist of layers of unidirectional yarn or fabric, the direction of which is offset in each layer (Figure 2.28).

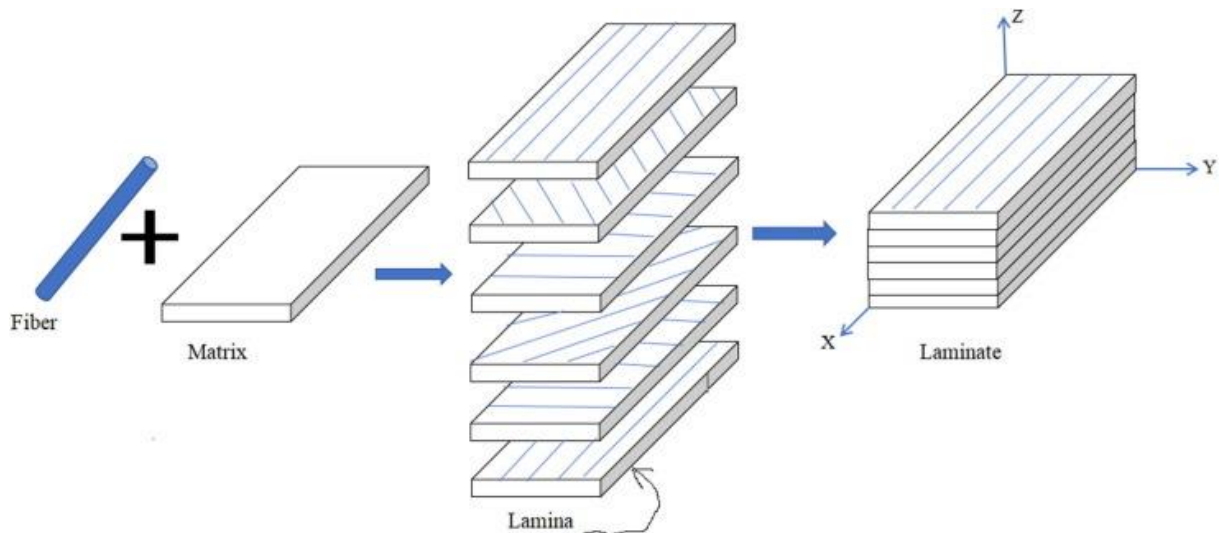


Figure 2.28: Composition of a laminate.

The designation of these laminates is generally carried out according to the following code:

1. Each layer is designated by a number indicating the value in degrees of the angle that the direction of the fibers makes with the reference x axis.
2. Successive layers are separated by a / if their angles are different.
3. Successive layers of the same orientation are designated by a Numerical index.
4. The layers are named successively from one face to the other. Brackets (or parentheses) indicate the beginning and end of the code.

The designation depends on the chosen axis system. An example is given in Figure 2.29.

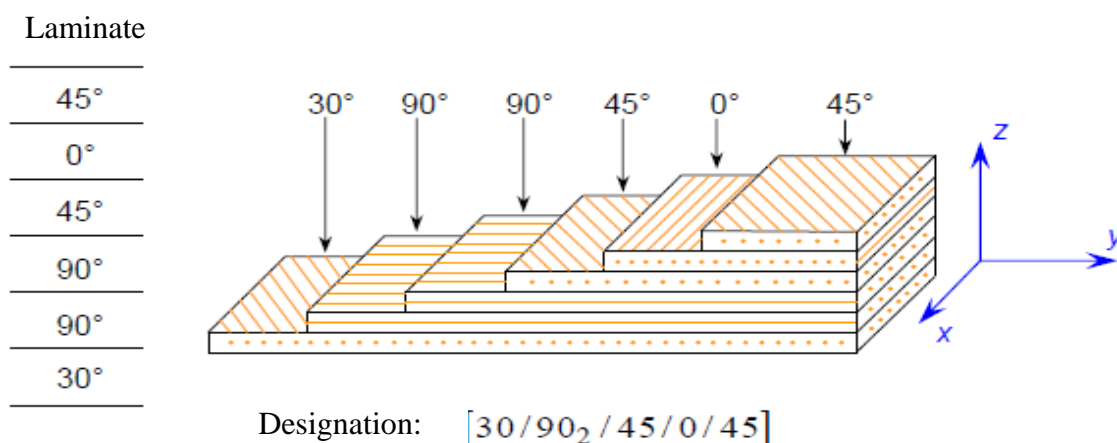


Figure 2.29: Laminate designation.

a. Positive and negative angles

When layers are oriented at angles equal in absolute values, but of opposite signs, the + or – signs are used. The convention for positive or negative angles depends on the axis system chosen: an inversion may appear depending on the choice made (figure 2.30).

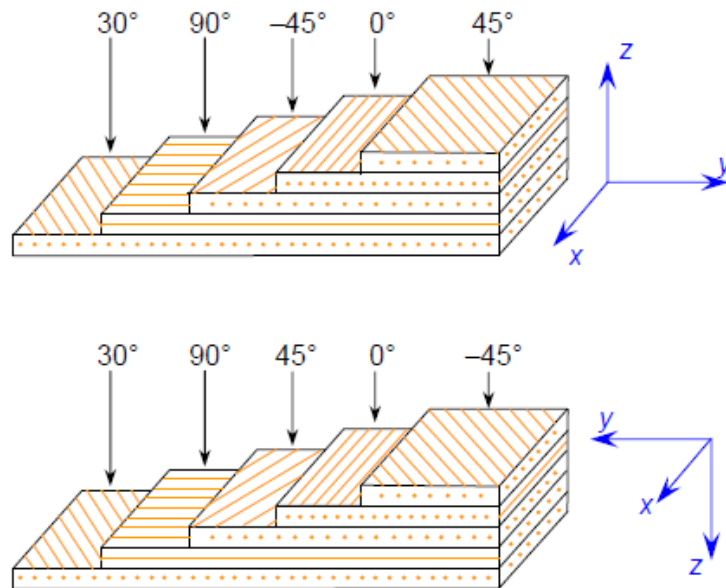


Figure 2.30: Sign convention for the designation of laminates.

Below we give some examples of laminate designations.

Laminate	Designation
<u>0°</u>	
<u>30°</u>	
<u>-30°</u>	[±45 / ∓30 / 0]
<u>-45°</u>	
<u>45°</u>	
<u>30°</u>	
<u>-60°</u>	
<u>-60°</u>	[45 / 0 / -60 ₂ / 30]
<u>0°</u>	
<u>45°</u>	

Laminate	Designation
$\begin{array}{c} \hline 0^\circ \\ \hline \hline -45^\circ \\ \hline -45^\circ \\ \hline \hline 45^\circ \\ \hline 45^\circ \\ \hline \end{array}$	$[45_2 / -45_2 / 0]$
$\begin{array}{c} \hline 0^\circ \\ \hline \hline -45^\circ \\ \hline 45^\circ \\ \hline \hline 45^\circ \\ \hline -45^\circ \\ \hline -45^\circ \\ \hline \hline 45^\circ \\ \hline \end{array}$	$[\pm \mp \pm 45 / 0]$ Or $[45 / -45_2 / 45_2 / -45 / 0]$

b. Symmetrical laminates

A laminate is symmetrical if its middle plane is the plane of symmetry. Its designation then requires only half of the successive layers.

If the laminate has an even number of layers, the designation begins on one face and ends at the plane of symmetry. An S index indicates that the laminate is symmetrical.

For example:

$\begin{array}{c} \hline 90^\circ \\ \hline 45^\circ \\ \hline 45^\circ \\ \hline 0^\circ \\ \hline 0^\circ \\ \hline 45^\circ \\ \hline 45^\circ \\ \hline 90^\circ \\ \hline \end{array}$	$[90 / 45_2 / 0]_S$
---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	---------------------

If the laminate has an odd number of layers, the designation is similar to the previous one, with the middle layer upper lined. For example:

$$\begin{array}{c}
 \hline 90^\circ \\
 \hline 45^\circ \\
 \hline 45^\circ \\
 \hline 0^\circ \\
 \hline 45^\circ \\
 \hline 45^\circ \\
 \hline 90^\circ \\
 \hline
 \end{array}
 \quad [90 / 45_2 / \bar{0}]_S$$

c. Sequences

The repetition of sequences may be indicated by an index indicating the number of times a sequence is successively repeated.

For example:

$$\begin{array}{c}
 \hline 0^\circ \\
 \hline 45^\circ \\
 \hline 90^\circ \\
 \hline 0^\circ \\
 \hline 45^\circ \\
 \hline 90^\circ \\
 \hline 90^\circ \\
 \hline 45^\circ \\
 \hline 0^\circ \\
 \hline 90^\circ \\
 \hline 45^\circ \\
 \hline 0^\circ \\
 \hline
 \end{array}
 \quad [(0 / 45 / 90)_2]_S$$

ou $[0 / 45 / 90]_{2S}$

$$\begin{array}{c}
 \hline 0^\circ \\
 \hline 45^\circ \\
 \hline 90^\circ \\
 \hline 0^\circ \\
 \hline 45^\circ \\
 \hline 90^\circ \\
 \hline 60^\circ \\
 \hline 30^\circ \\
 \hline 60^\circ \\
 \hline 30^\circ \\
 \hline 60^\circ \\
 \hline 30^\circ \\
 \hline
 \end{array}
 \quad [(30 / 60)_3 (90 / 45 / 0)_2]$$

The alternation $[0/90]_n$ is often referred to as cross-ply.

d. Hybrid laminates

Hybrid laminates are made up of successive layers containing fibres of different types. It will then be necessary to mention them in the designation. For example, for laminates with the same resin:

0 ° V	
45 ° C	
-45 ° C	
90 ° C	
90 ° C	
-45 ° C	
45 ° C	
0 ° V	
[0 _V / ±45 _C / 90 _C] _s	
0 ° K	
0 ° K	
45 ° V	
90 ° V	
45 ° V	
0 ° C	
0 ° C	
[0 _{2C} / (45 / $\overline{90}$) _{SV} / 0 _{2K}]	

with V: glass, C: carbon, K: Kevlar

Hybrid laminates allow for better performance by making the best use of the properties of the various available fibers. Among the different hybrids, we can distinguish:

- ✓ Interlayer hybrids, consisting of a series of layers, each of a different nature;
- ✓ Intralayer hybrids, consisting of a sequence of identical layers, each layer being made up of different reinforcements;

Metal layers can also be intercalated between the layers.

2.6.2.2. General structure of a laminate

In the general case, the reinforcement of each layer will be of various types: yarns, rovings, mats, fabrics, glass fibers, carbon fibers, etc. Each layer must then be designated by indicating

the nature of the fibers, the type of reinforcement: yarns, mat, fabric with an indication of the proportion of fibers in the warp and weft direction.

The choice of the nature and sequence of the layers will depend on the use of the composite material, adapting it as best as possible to the imposed stress field:

- ✓ Unidirectional layers have good mechanical performance in the direction of the fibers;
- ✓ Mats have low tensile strength and should be reserved for compressed zones: for example (Figure 2.31) unidirectional layers in the tensile zone, mat in the compression zone of a beam subjected to bending stress;
- ✓ Cross-lamination will be sensitive to interlaminar delamination;
- ✓ A lamination with at least three fiber directions will be necessary to have a pseudo-isotropy in the plane of the laminate.

Finally, it is interesting to note that a symmetrical lamination will generally guarantee a good flatness of the laminate after demoulding.

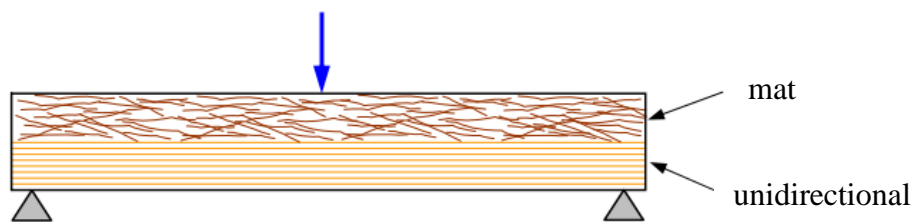


Figure 2.31: Beam subjected to bending.

2.6.3. Sandwiches

The principle of the sandwich technique consists of applying to a core (made of a material or a light structure with good compression properties) two “sheets”, called skins, with good tensile characteristics. The objective of such a process is to constitute a structure that combines lightness and rigidity.

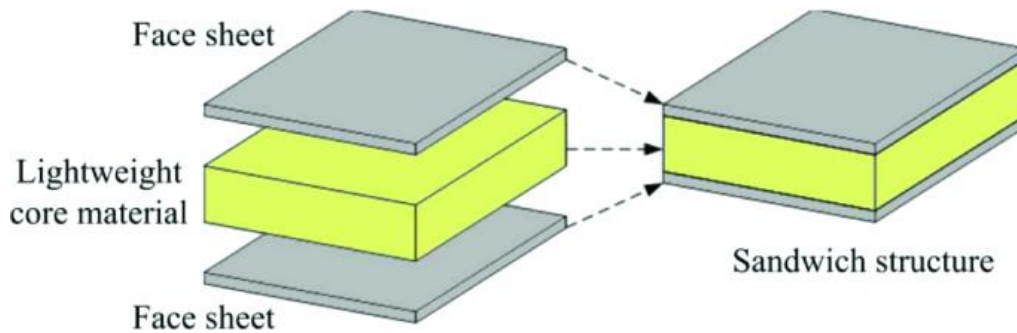


Figure 2.32: Sandwich composite.

Generally, the choice of materials is made with the initial objective of having a minimum weight, then taking into account the conditions of use (thermal conditions, corrosion, price, etc.).

The most commonly used materials are:

For solid cores (figure 2.33):

- ✓ Balsa or cellular wood;
- ✓ Various cellular foams;
- ✓ Resins loaded with hollow glass microspheres, called syntactic foams;

For hollow cores (figure 2.34), mainly honeycomb and profiles:

- ✓ Light metal alloys;
- ✓ Kraft paper (resin coated or not);
- ✓ Polyamide paper, such as Nomex paper;

Mixed cores can be used.

The skins are most often laminates (glass, carbon, Kevlar) or sheets of light alloys.

For sandwich structures to fully play their role, it is necessary to ensure perfect bonding of the core-skin assembly, so as to distribute the forces between the core and the skins. The assembly is carried out by bonding using resins compatible with the materials present.

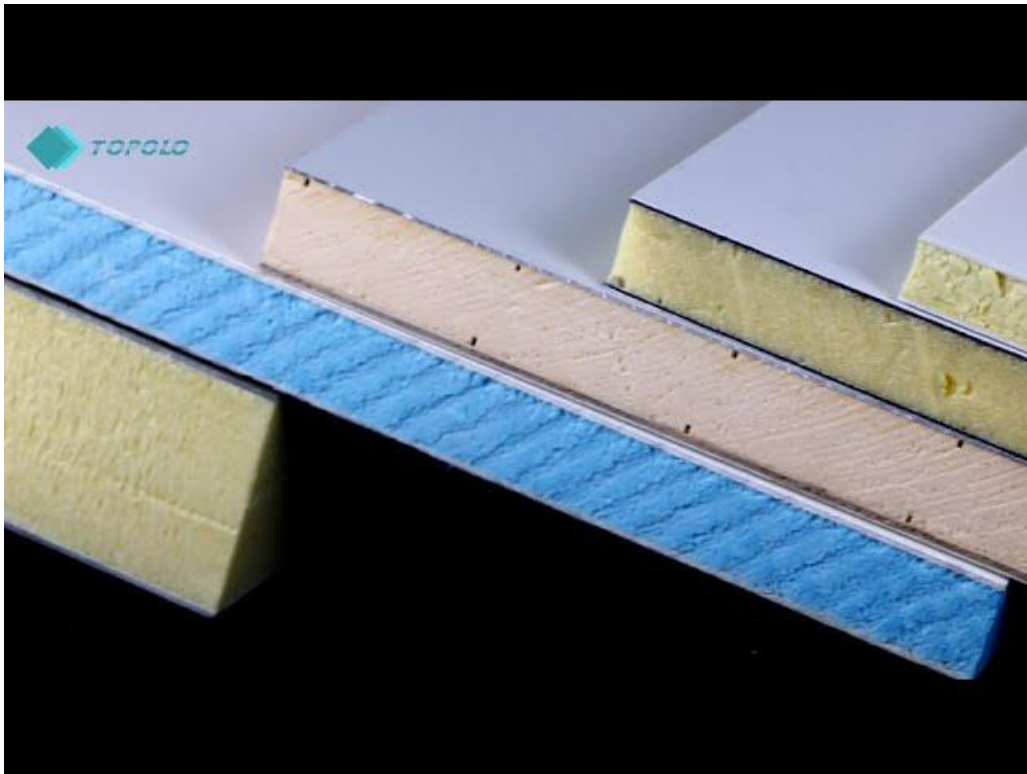


Figure 2.33: Solid core sandwich materials.

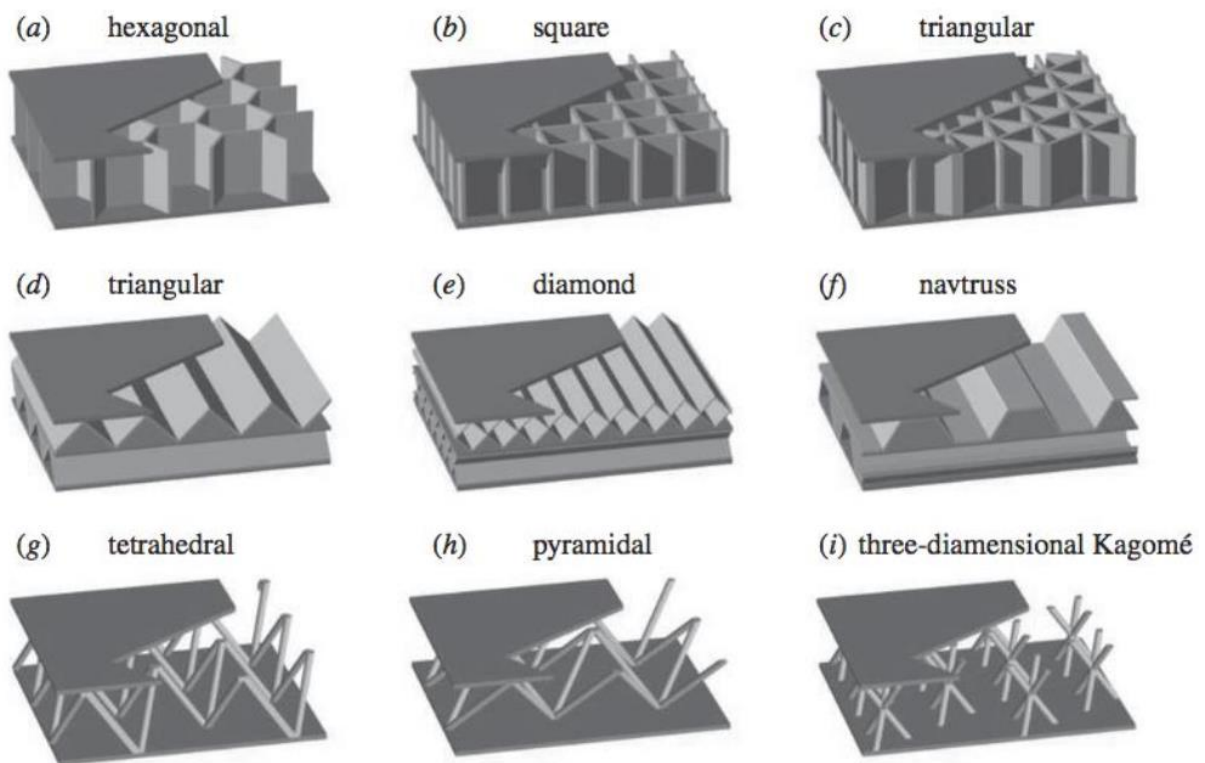


Figure 2.34: Hollow core sandwich materials.

2.6.4. Other architectures

Other composite material architectures can be schematically classified into: reinforced plastics and volume composites.

2.6.4.1. Reinforced plastics

These materials consist of resins or matrices into which reinforcements of the following type are introduced:

- ✓ Short fibers,
- ✓ Solid or hollow beads,
- ✓ Powders: metallic, graphite.

The reinforcements generally make it possible to increase the modulus of elasticity by a factor of 2 to 5. The mechanical behaviour of these materials can be homogenized, and its study is reduced to that of a usual isotropic material.

2.6.4.2. Volumetric composites

Volumetric composites were introduced for specific aeronautics needs. They are developed from volume weaves. These materials are very expensive. In addition to specific interests, they make it possible to obtain very high mechanical characteristics, with a behaviour that is substantially isotropic in volume.

2.7. Process of studying the mechanical behaviour of composite materials

The study of the implementation of composite materials has shown the importance of laminates and sandwich materials. The architecture of these materials now allows us to outline the main points of the study of their mechanical behaviour.

This study will consist of two phases:

1. The study of the mechanical behaviour of each layer, sometimes called micromechanical or microscopic behaviour of the composite material. This study is quite often referred to as microanalysis of the composite material.

2. The study of the overall behaviour of the material made up of several layers, and generally referred to as macroscopic behaviour of the composite material or behaviour of the laminate.

These two studies having been carried out; the global mechanical behaviour of a composite structure will then be analysed by adapting the classic tools of structural calculation to the macroscopic behaviour of composite materials.

The analysis of simple structures (beams and plates) can generally be approached by an analytical method, while the study of complex structures requires the use of the finite element method.

Chapter 3: Composite manufacturing technology

3.1. Introduction

The objective of this chapter is not to make an in-depth study of the technology for manufacturing composite materials. It simply aims to identify the principles of different manufacturing processes.

Most manufacturing methods consist of developing composite material parts by successive layers comprising matrix and reinforcement. This general technique called stacking (stratification), leading to the development of laminates.

3.2. Manufacturing process of composite materials

3.2.1. Castings without pressure

Cold moulding methods without the intervention of a press are the simplest methods to implement. They require a minimum of equipment and therefore depreciation. This ease has been at the origin of the success of fiberglass composite materials, in industry and crafts.

These methods allow the production of parts in small and medium series, without restriction of shapes and dimensions. Although the proportion of fibers can vary, it remains limited. The parts have a single smooth face, reproducing the appearance of the mould. Finally, the quality of the moulded part depends to a large extent on the know-how of the moulder.

3.2.1.1. Contact moulding (casting)

Before moulding, the mould is coated with a mould release agent, then generally with a thin layer of surface resin, often coloured, called a “gel coat”.

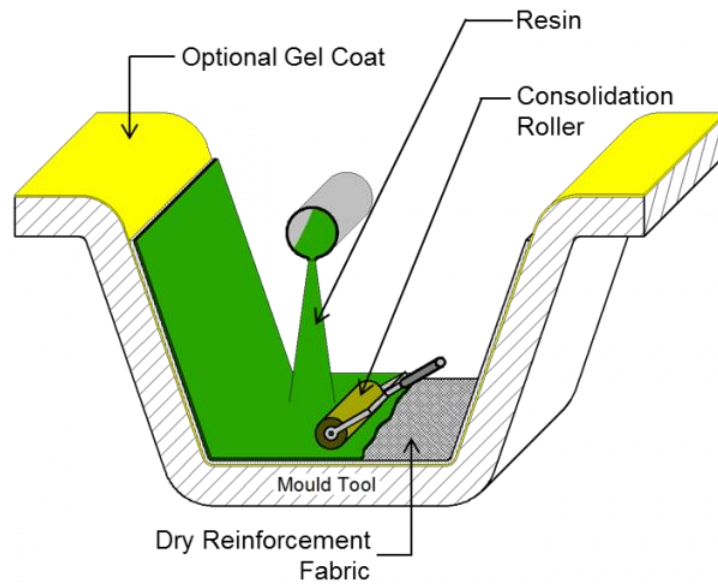


Figure 3.1: Contact moulding principle.

The moulding is then carried out according to the following operations:

1. The mould is coated with catalysed and accelerated resin, using a brush or roller.
2. The reinforcement: mat, fabric, etc., is placed in the mould. Various types of reinforcements can be used depending on the different parts of the desired piece. The reinforcements must then be superimposed.
3. The reinforcement is then impregnated with the matrix, then a bubble removal is carried out with a grooved roller.
4. After gelling of the first layer, the following layers are applied, using the same technique. Inserts can be placed between these layers: tubes, screws, nuts, reinforcements, etc.
5. The demoulding is then carried out after a time that depends on the resin and the temperature (of the order of 10 hours).
6. The polymerization is then carried out in an ambient environment for several weeks. This polymerization can possibly be accelerated by baking (for example 5 to 10 hours, at around 80 °C).
7. After polymerization, we proceed to finishing the part: trimming, sanding, possibly painting, etc.

3.2.1.2. Simultaneous spray (projection) moulding

Moulding is carried out by simultaneous projection of chopped fibers and catalysed resin onto a mould. The equipment to be projected consists of a roving cutting machine and a gun projecting the resin and chopped fibers, the assembly operating by compressed air. The layer of fibers impregnated with resin is then compacted and freed from bubbles using a grooved roller.

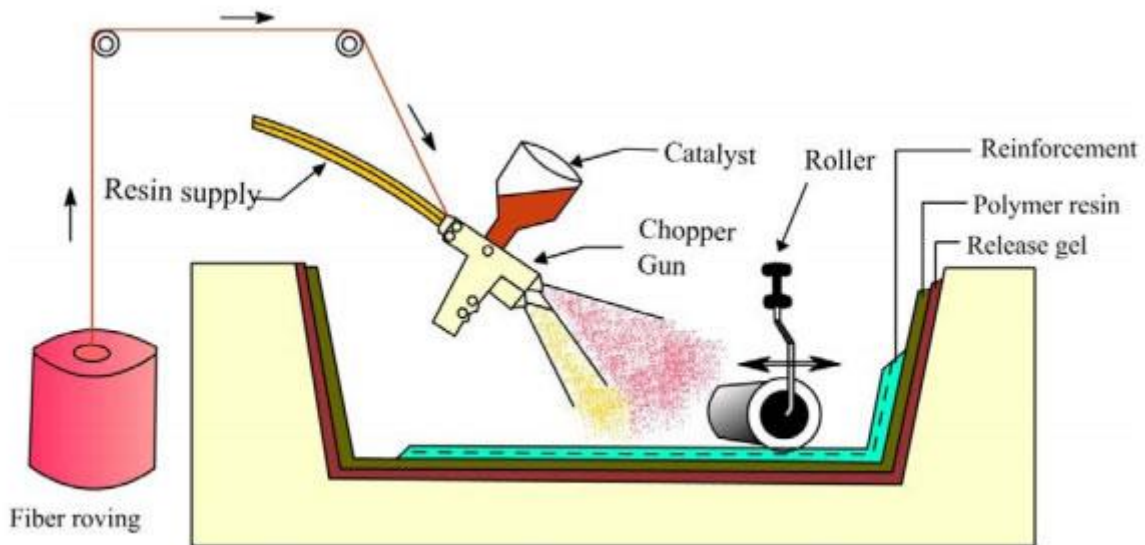


Figure 3.2: Principle of simultaneous projection moulding.

Projection moulding makes it possible to obtain large series of parts, with a low-cost price. The reinforcement is, however, limited to chopped fibers, and the mechanical characteristics of the material remain average. It is possible to obtain two smooth faces by using a mould and counter-mould, loaded separately, then joined. This process also reserves the possibility of interposing a layer of fabric between the two, and then makes it possible to obtain parts with better mechanical characteristics.

3.2.2. Vacuum moulding (casting)

Vacuum casting involves using vacuum and atmospheric pressure simultaneously. After coating with gel coat, the reinforcement is placed on a rigid mould, then the matrix is cast. The counter-mould, covered with a membrane ensuring sealing (rubber sheet, nylon, etc.), is then fitted. A vacuum pump creates a depression through the mould and the porous counter-mould, which spreads and de-bubbles the resin. The counter-mould can possibly be limited to

the sealing membrane alone. This casting process is suitable for the manufacture of parts in small and medium series. It allows good mechanical qualities to be obtained, thanks to a uniform proportion of resin and a reduction in air inclusions. In the case of using a rigid counter-mould, a beautiful surface appearance is obtained on both sides. However, production rates are quite slow.

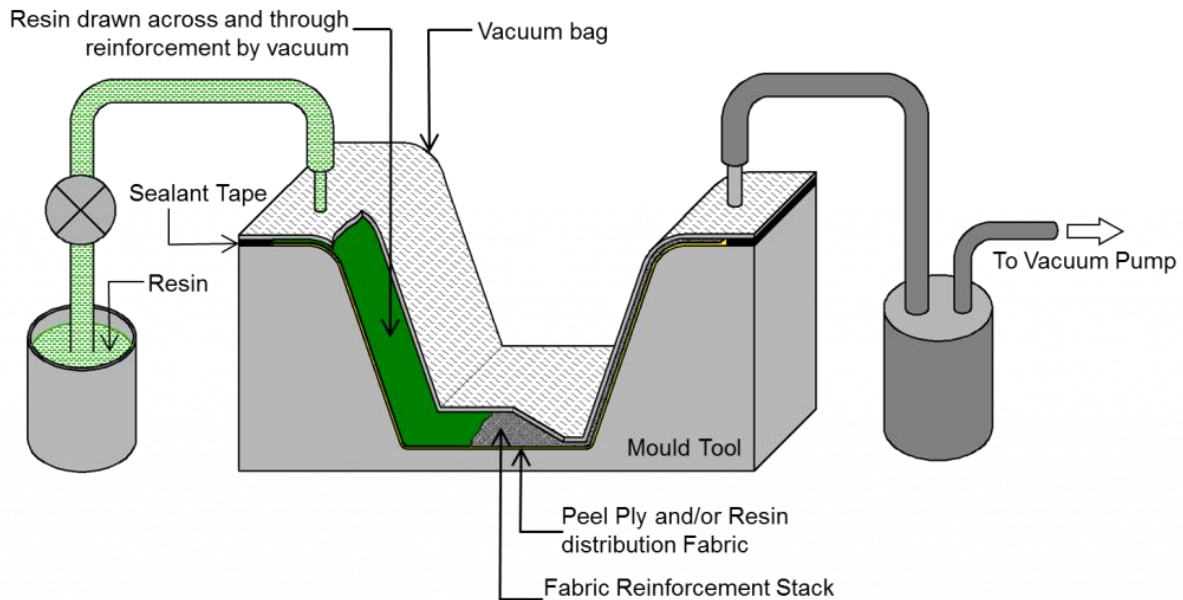


Figure 3.3: Vacuum moulding

3.2.3. Compression moulding

3.2.3.1. Resin injection moulding

Moulding by injection of resin under pressure consists of impregnating a reinforcement placed inside a very rigid and closed mould and counter-mould assembly. The automatic feeding of the resins eliminates their handling. The proportion of reinforcement can be high, resulting in parts with high mechanical characteristics. This moulding process is suitable for producing deep parts and complicated shapes.

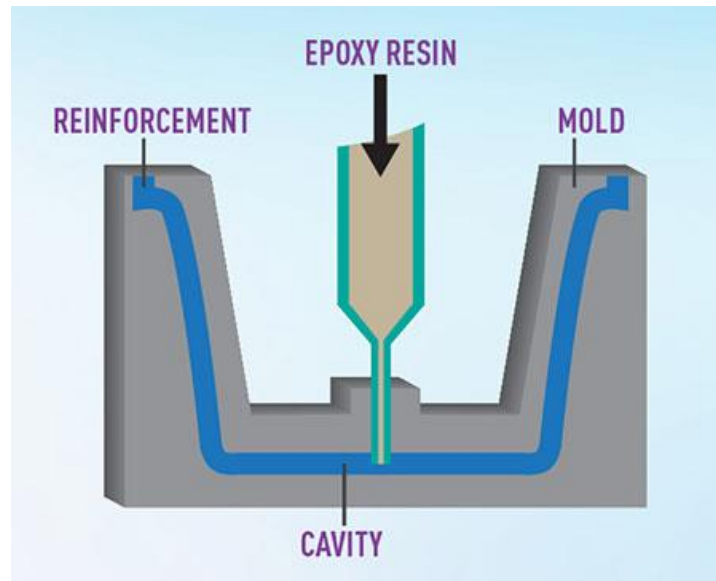


Figure 3.4: Resin injection moulding

3.2.3.2. Cold compression moulding

Moulding is performed at low pressure (<5 bars) without heating the mould, using the exothermic polymerization of the resin. The heat energy accumulated by moulding the parts is then sufficient to maintain the mould at temperatures of 50 to 70 °C, in permanent operation.

Mold and counter-mould are coated with release agent and gel-coat. Then the reinforcement and the matrix are placed on the mould. The mould/counter-mould assembly is closed, then pressed. The polymerization time is linked to the type of resin, the catalyst and the temperature reached by the mould in continuous production mode.

This moulding process is suitable for the manufacture of medium-series parts (4 to 12 parts per hour). The investment (equipment and mould) is less than the hot compression process. The low-pressure press is simplified. The moulds can be made by the transformer from composite materials. The parts have a beautiful surface appearance on each side. Productivity is lower than hot press moulding.

3.2.3.3. Hot compression moulding

This technique makes it possible to obtain parts in large series using hydraulic presses and heated metal moulds.

The reinforcement, consisting of cut-thread or continuous-thread mat, fabrics or preforms, is placed on the heating mould, previously coated with a release agent. Then the catalysed resin is poured in bulk onto the reinforcement. The mould is closed according to a cycle determined by lowering and pressing the counter-mould. The pressing time is linked to the polymerization time of the resin, a function of the reactivity of the resin and the thickness of the part. The mould is then opened, and the part ejected.

This moulding process makes it possible to obtain significant proportions of reinforcement, and consequently parts with good mechanical characteristics. The dimensions of the parts depend on the size of the press. The moulding pressure is of the order of 10 to 50 bars, the temperature of the moulds of the order of 80 to 150 °C. Manufacturing rates can reach 15 to 30 pieces per hour. They require a significant investment in equipment, press and mould.

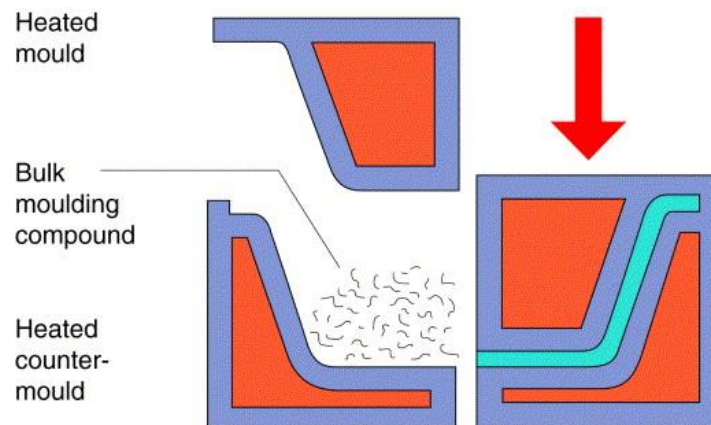


Figure 3.5: Compression moulding principle (heated).

3.2.3.4. Injection moulding

The injection moulding method is the most widespread method of implementing reinforced thermoplastics (other methods being extrusion, blow moulding, thermoforming, etc.).

Injection moulding is carried out on conventional presses used for injecting thermoplastic resins.

Granules containing the resin and the reinforcement (short fibers, beads, etc.) or pre-impregnated mats are extruded by an Archimedes screw. The matrix is fluidized by heating and injected under high pressure into a heated mould, where polymerization takes place.

The type of materials obtained is more generally called “reinforced plastics” than composite materials. Indeed, given the nature of the reinforcements (short fibers, spheres, etc.), the breaking stress and Young's modulus of the resins are multiplied by a factor of around 2 to 4. This technique is suitable for the production of parts in very large series.

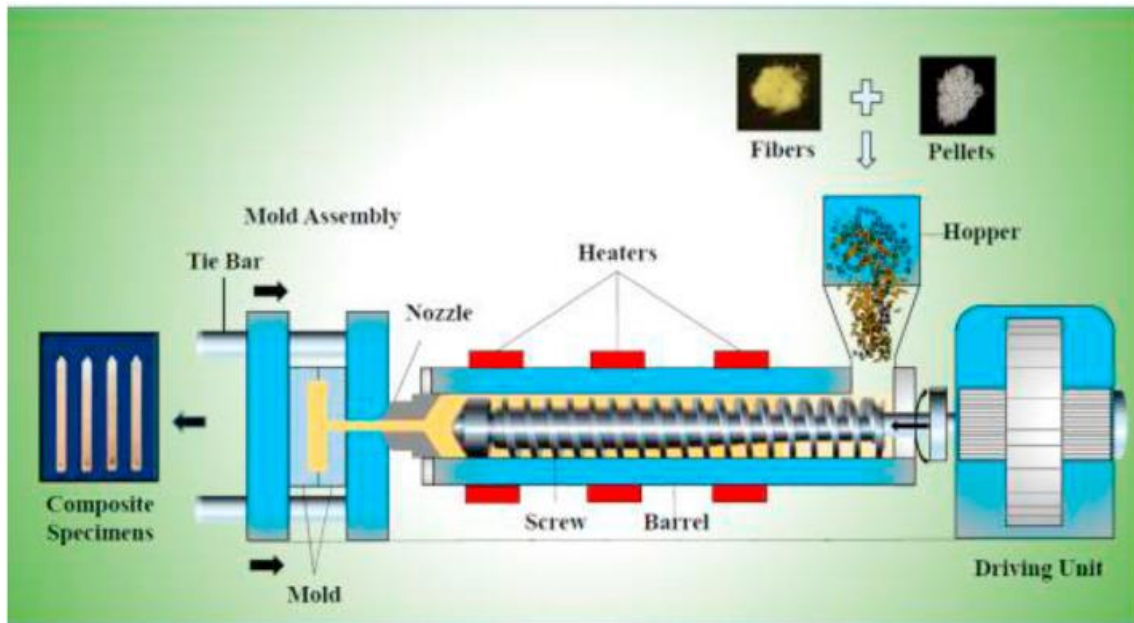


Figure 3.6: Injection moulding

3.2.4. Continuous moulding

Continuous moulding allows the manufacture of flat sheets, panels, sandwiches (figure 3.7), corrugated roof panels (figure 3.8), ribbed sheets, etc.

Schematically, this process can be separated into several phases.

1. A reinforcement impregnation phase: fibers, mats or fabrics. The catalysed resin and the reinforcement are transported on a release film (cellophane, mylar, polyethylene, etc.).
2. A shaping phase.
3. A polymerization phase, carried out in a tunnel-shaped oven (60 to 150 °C), the length of which depends on the temperature and the resin (15 to 50 m long).
4. A cooling and cutting phase.

In the case of the manufacture of flat sheets (figure 3.7), the shaping is simply carried out by adjusting the thickness of the sheet, by pressing between calendaring rollers.

In the case of corrugated panels, shaping occurs during polymerization (figure 3.8), using moving rollers.

The continuous moulding process can be fully automated, and then allows the continuous production of plates or panels. However, it requires a very significant investment in equipment.

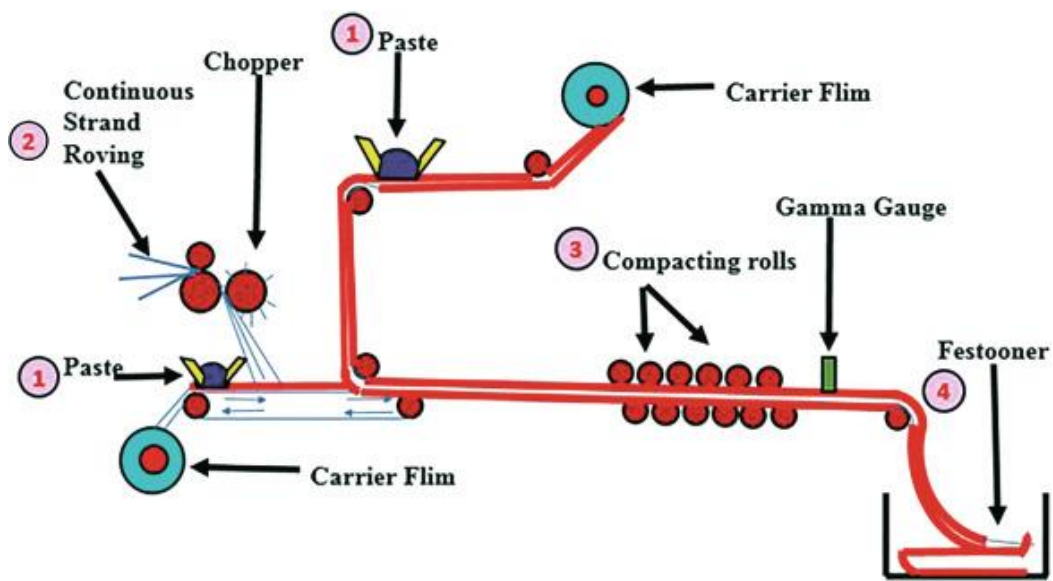


Figure 3.7: Continuous moulding of plates.

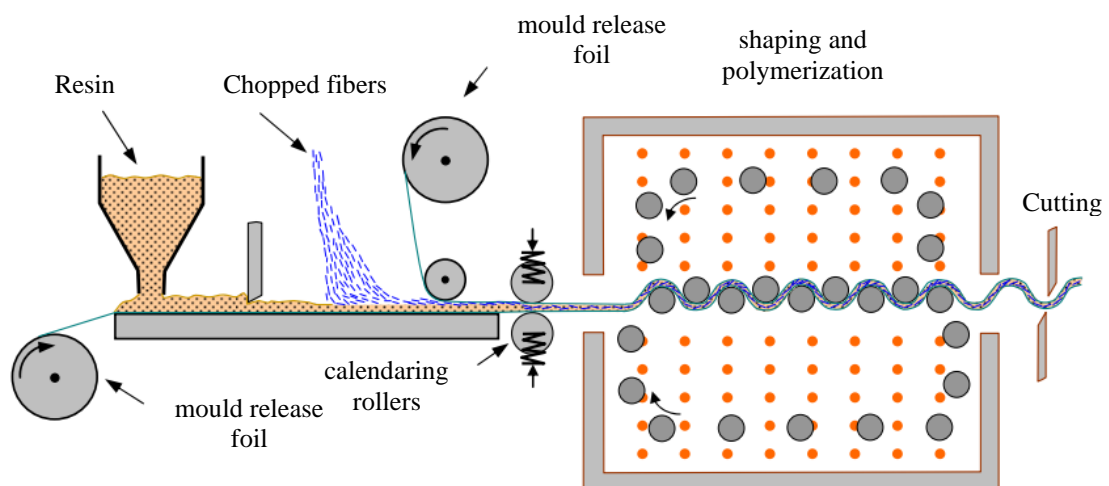


Figure 3.8: Continuous moulding of corrugated panels.

3.2.5. Pultrusion moulding

The pultrusion moulding process is used to manufacture profiles, straight or curved, with constant section, highly reinforced in the main direction.

In this technique (figure 3.9), the reinforcements: threads, rovings, ribbons, etc., pass through a bath of catalysed resin where they are impregnated. They then pass through a heated die in which the profile is shaped and the resin polymerized simultaneously.

This process is applicable to thermoplastic and thermosetting resins. The profiles obtained have high mechanical characteristics, given the possibility of obtaining high reinforcement proportions of up to 80% by volume. The process is suitable for production of fairly large series (speed of up to 20 m/h). It requires a significant investment in equipment. Examples of manufacturing: fishing rods, various profiles, stiffeners, etc.

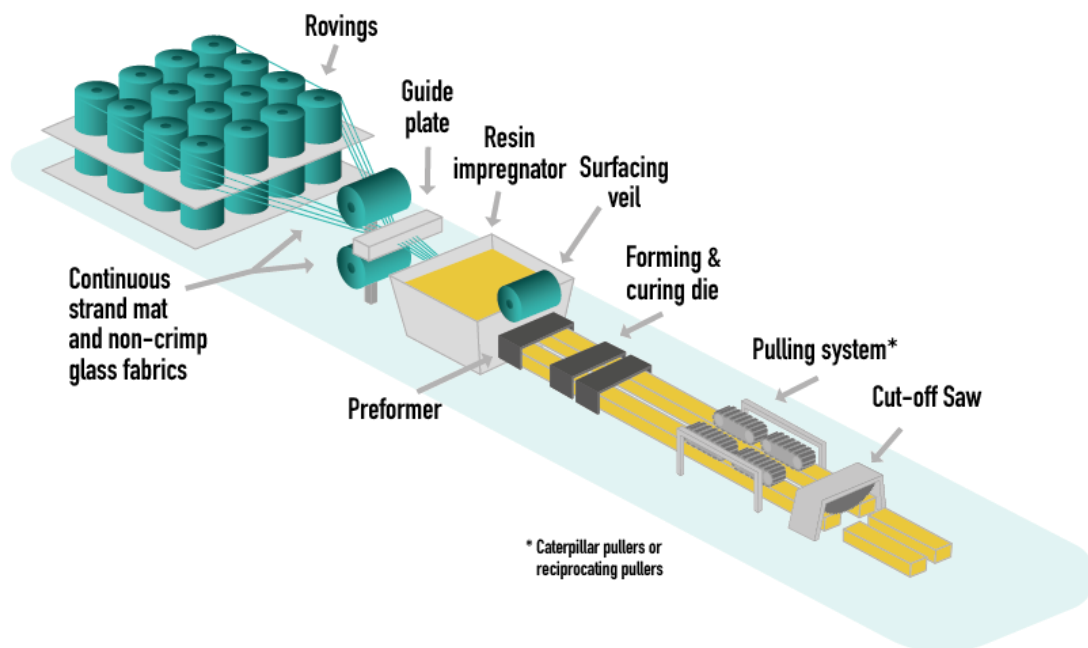


Figure 3.9: Pultrusion moulding.

3.2.6. Centrifugation moulding

This technique is reserved for the moulding of revolution parts, in particular tubes, pipes, tanks, etc. It is an extrapolation (figure 3.10) of the technique for manufacturing cast iron or centrifuged concrete pipes.

The revolution mould, coated with a release agent, is rotated (at approximately 2,000 rpm). After possible deposition of gel coat, the following are introduced simultaneously continuously:

- ✓ reinforcement: chopped fibers or cut roving;
- ✓ catalysed and accelerated resin (epoxy resins, polyesters, etc.) hardening in the cold.

The reinforcement is impregnated with resin by centrifugation. Stratification is carried out by successive passes of the resin and reinforcement feed nozzle. Polymerization is carried out at room temperature, or possibly accelerated in an oven.

A reinforcement in the form of a roll (mat, fabric, etc.) can be introduced discontinuously before rotation of the mould. The resin is then introduced during centrifugation.

After polymerization, the cylindrical part is extracted from the mould, the removal of the resins allowing demoulding. This technique makes it possible to obtain a beautiful surface appearance on the outside, with a well-calibrated diameter and thickness of the parts. This manufacturing process requires high-precision equipment and very good mould calibration.

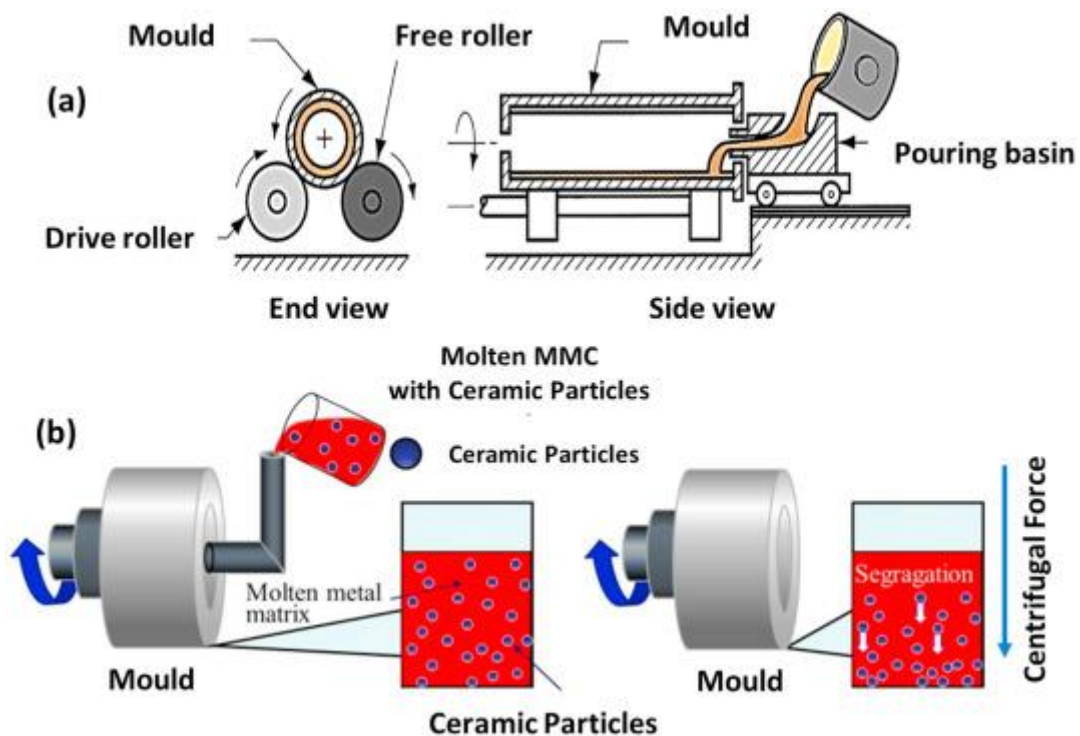


Figure 3.10: Centrifugation moulding

3.2.7. Filament winding moulding

3.2.7.1. Principle

The reinforcement (continuous thread, ribbon, etc.) impregnated with catalysed resin is wound with slight tension, on a cylindrical or rotating mandrel.

This type of moulding is well suited to cylindrical and spherical surfaces, and allows advanced design of the parts. The laminates obtained can include high proportions of reinforcement (up to 80% by volume), thus allowing high mechanical characteristics to be obtained. The investment in equipment is very high.

Depending on the relative movements of the mandrel and the reinforcement supply system, various types of windings (and therefore stratifications) are obtained. There are: circumferential winding, helical winding, polar winding.

3.2.7.2. Circumferential winding

The winding is carried out at 90° to the mandrel axis and provides high tangential resistance. To obtain satisfactory longitudinal strength, it is necessary to insert layers of unidirectional fabrics in the axial direction of the mandrel.

This type of winding is rarely used.

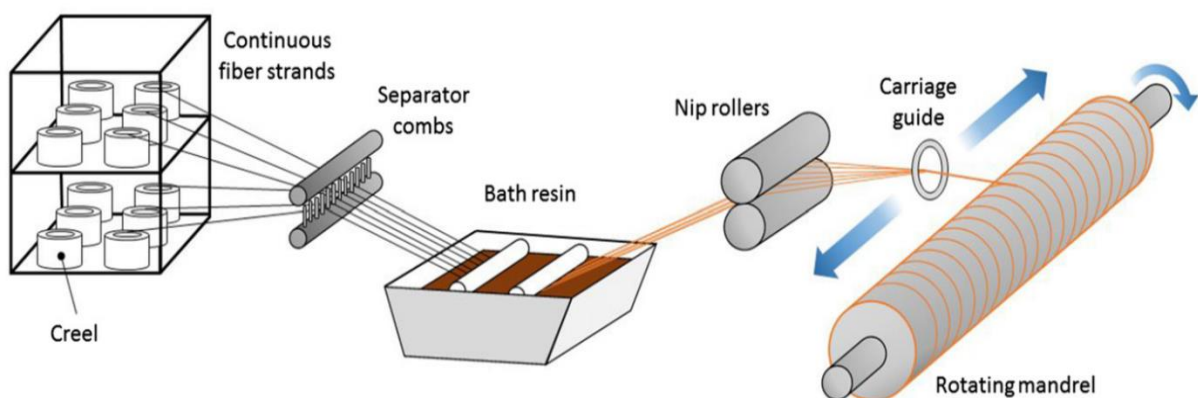


Figure 3.11: Circumferential winding principle.

3.2.7.3. Helical winding

3.2.7.3.1. Discontinuous winding

The direction of winding of the threads is inclined relative to the axis of the mandrel by an angle whose value is determined by the relative movement of the thread guide with respect to the rotation of the mandrel (figure 3.12). The value of the angle is chosen according to the desired ratio between the tangential resistance and the transverse resistance. The thread sheet is regularly distributed and stratified over the entire surface of the mandrel by alternating movements of the thread guide parallel to the axis of the mandrel. This type of winding gives great freedom for the angular arrangement of the threads. In particular, it allows successive layers with different angles to be produced. This winding process has many applications for the manufacture of large parts such as containers, the manufacture of rocket casings, torpedoes, oil drilling tubes, gas cylinders, etc.

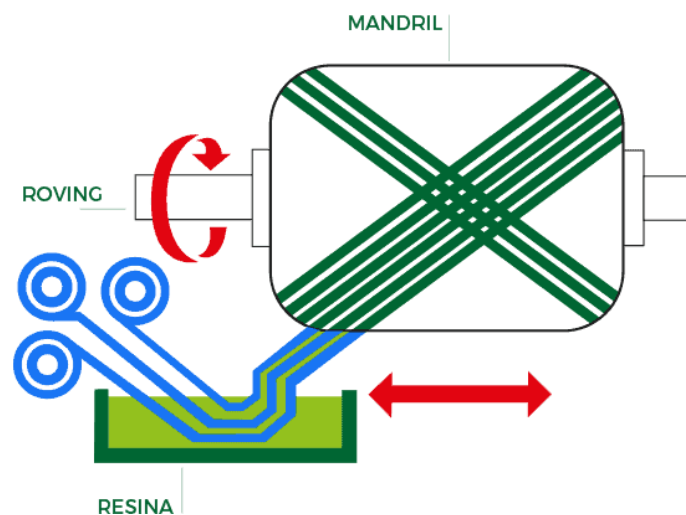


Figure 3.12: Principle of discontinues helical winding

3.2.7.3.2. Continuous winding

Continuous winding (Figure 3.13) enables the industrial manufacture of high-performance tubes and pipes of various diameters and long lengths.

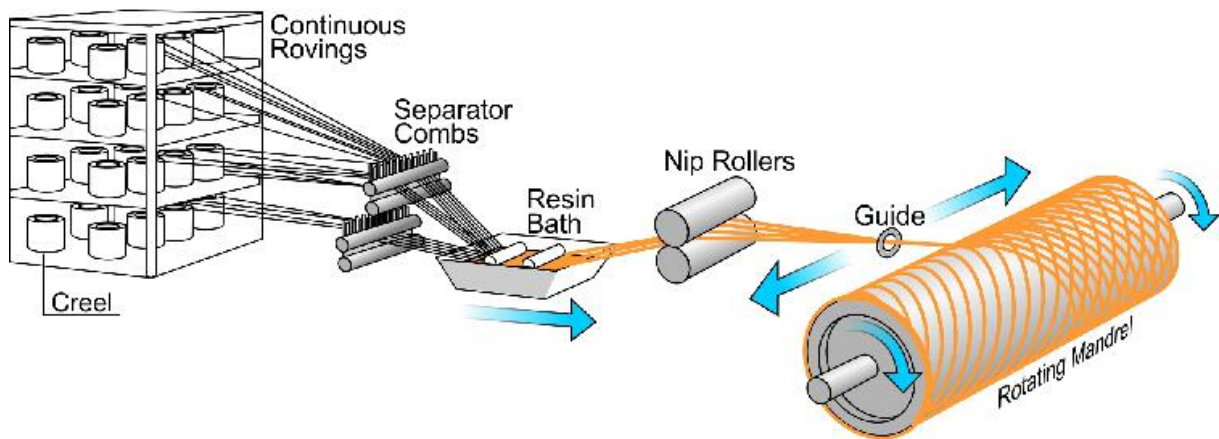


Figure 3.13: Continuous helical winding.

3.2.7.4. Polar winding

Polar winding allows the manufacture of parts with spherical ends without winding discontinuity (Figure 3.14). In this type of winding, the mandrel must have three degrees of freedom in rotation, allowing 3 simultaneous or non-simultaneous rotations to be controlled. This technology is used to manufacture high-pressure tanks, rocket engine tanks, space equipment, etc.



Figure 3.14: Polar winding.

3.2.7.5. Mandrels

The mandrels for filament winding must allow demoulding. They can be:

- ✓ made of metal, wood, etc., in one piece or in several removable elements;
- ✓ made of a material with a low melting point;
- ✓ made of a soluble material: for example, grains of sand agglomerated in a water-soluble binder;
- ✓ made of an inflatable elastomer.

3.2.7.6. Applications

The applications of filament winding have been identified in the previous paragraphs. Generally speaking, this implementation process is used to manufacture parts with rotational symmetry: pipes, tubes, tanks, gas cylinders, cylindrical casings, etc. Large parts can be produced: containers, silos, etc. The dimensions of the parts are limited by the type of machine used. The advantage of filament winding also lies in the possibility of computer-aided automation. The filament winding process is also applicable to parts without rotational symmetry: helicopter and turbine blades, tanks with rectangular sections, etc.

3.3. Use of semi-products

3.3.1. Introduction

The implementation of semi-finished products (prepregs, compounds) uses the same techniques as moulding from threads or fabrics, impregnation of which with resin is carried out at the time of implementation. We have separated the presentation, taking into account the specific character of these semi-finished products: ease of handling, possibility of advanced automation of the implementation processes from these products, etc. It is therefore conceivable to think that the use and development of prepregs and compounds of various types, associated with computer-aided design and robotization, will make it possible to increasingly penetrate industrial markets: automobile construction, shipbuilding, armament, etc

3.3.2. Prepregs

3.3.2.1. Principle

Prepregs are products sold in the form of rovings, ribbons, fabrics, etc., impregnated with resin generally dissolved in a solvent.

The resins can be phenolic, polyester, epoxy resins, polyimides, etc. The volume percentage of fibers is high (50 to 80%) in order to obtain composites with high mechanical performance.

The impregnated roving or “stratipreg” is mainly intended for filament winding. The impregnated fabrics are generally thin (of the order of 1/10 mm), and therefore of low surface weight: 100 to 300 g/m². However, for the needs of shipbuilding, certain thick prepregs have been developed to simplify implementation.

3.3.2.2. Elaboration

Prepregs are manufactured on a vertical or horizontal machine, following the basic diagram in Figure 3.15. The thread or fabric is unwound at slow speed (1 to 10 m/min), and passes into a tank containing an impregnating resin diluted in a solvent. At the outlet, scrapers and spin rollers remove excess resin. Indeed, we have indicated that one of the advantages of prepregs is that they have a low resin content. The impregnated thread or fabric then passes through an oven, where most of the solvent evaporates, and where polymerization begins. At the outlet of the oven, the prepreg is cooled by ventilation, so as to stop the polymerization, which must remain unfinished. The fabric is then:

- ✓ either rolled between two polyethylene sheets to be delivered in rolls,
- ✓ or cut and stacked between two polyethylene sheets to be delivered in plates.

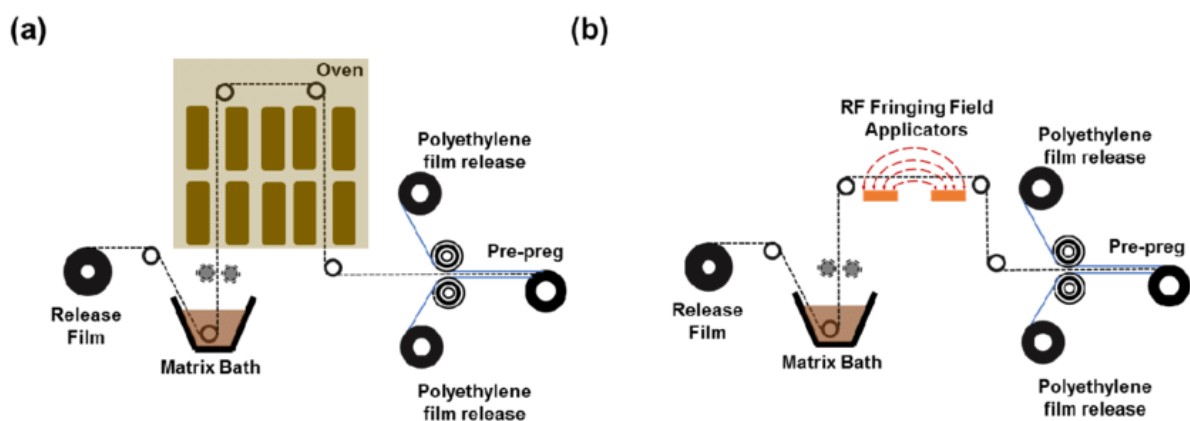


Figure 3.15: Schematic diagram of the production of a prepreg.

3.3.2.3. Features of prepregs

The main advantages of prepregs are:

- ✓ high mechanical performance of the laminate obtained, resulting from the high proportion of fibres;
- ✓ improved working and safety conditions, due to the elimination of resin handling and the release of toxic vapours;
- ✓ possible automation by draping, using machines or robots, making it possible to reduce the cost prices of finished products.

3.3.2.4. Implementation

Prepregs can be implemented:

- ✓ by hot compression, then curing;
- ✓ by vacuum moulding, with curing in heated moulds.

During implementation, the prepregs must be cut to the dimensions of the parts to be produced. As a result, only simple shapes can be produced. After placing the prepregs in the mould, under the action of heat, the resin becomes fluid again, then hardens to obtain the final shape.

3.3.3. Moulding compounds

Moulding compounds are semi-finished products delivered in rolls or sheets with a thickness of approximately 3 mm, generally composed of polyester resin-glass fibers, placed between two polyethylene films. They are sometimes considered as prepregs, but their use clearly differentiates them. Among these compounds, the most used are:

- ✓ SMC: Sheet Moulding Compounds,
- ✓ HMC: High Moulding Compounds.

Various other types of compounds exist and are developed for each specific use. One of the advantages of polyester compounds is their very short polymerization time (a few minutes), hence the possibility of using them in large series production.

3.3.3.1. Principle

SMC is a laminated moulding compound, with a relatively low proportion of fibers (30% by weight), and a high proportion of fillers (50% by weight) in the resin. The fibers can be (figure 3.16):

- ✓ either chopped fibers (generally 25 to 50 mm), randomly distributed in the plane of the sheet (SMC-R);
- ✓ or unidirectional chopped fibers (SMC-D);
- ✓ or unidirectional continuous fibers (SMC-C).

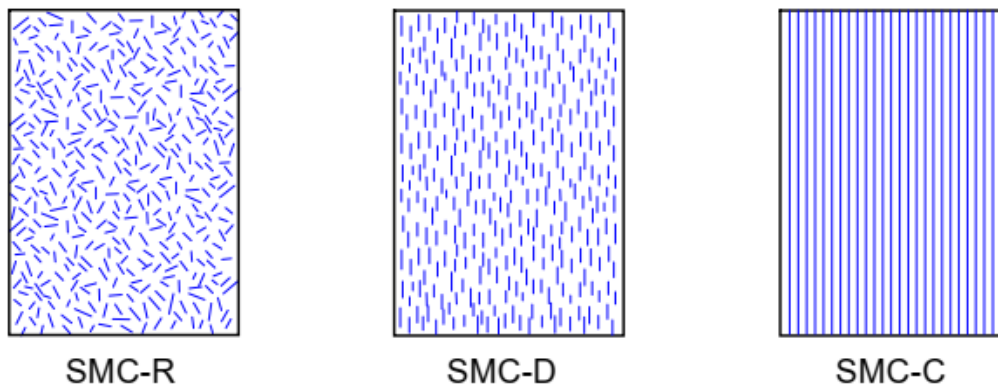


Figure 3.16: Different distributions of fibers in SMCs.

There are also mixed SMCs: for example, SMC-D/R (unidirectional chopped fibers and randomly distributed), SMC-C/R (unidirectional continuous fibers and randomly distributed chopped fibers).

The fibers can be of various types: glass, carbon, Kevlar, or mixed. By reducing the proportion of fillers in the resin, it is possible to increase the proportion of fibres from 30 to 50% by weight. The compound is then called HMC and has high mechanical characteristics.

3.3.3.2. Elaboration

The principle of compound preparation is given in Figure 3.17. The resin in the form of paste is conveyed on a polyethylene sheet. The fibers are placed on the resin, then covered by a second sheet of polyethylene. The whole passes through a compacting system, to then be rolled up.

The polymerization of the resin is delayed by the addition of magnesium oxide. The compound then appears in the form of soft and sticky sheets.

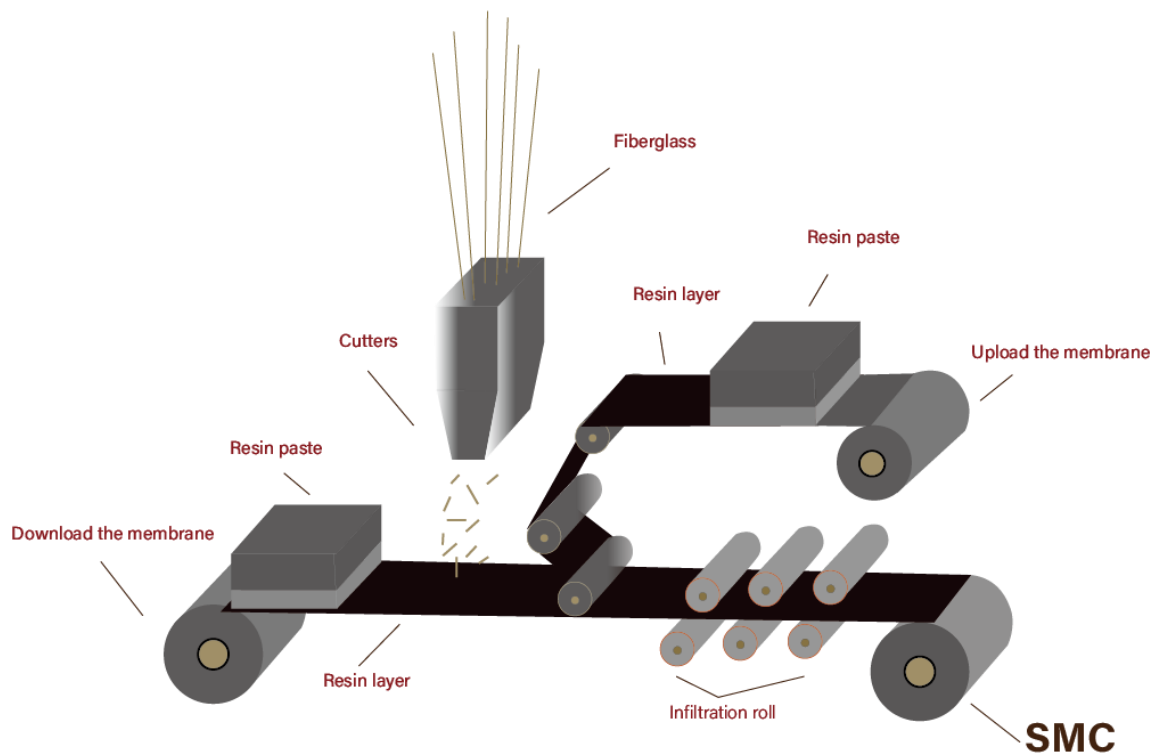


Figure 3.17: Principle of elaboration an SMC.

3.3.3.4. Implementation

Moulding is performed using traditional compression moulding techniques. “Blanks” of compounds are cut from the sheets and placed between the mould and the counter-mould. The shape of the blanks does not necessarily have to be adapted to the dimensions of the mould. In fact, unlike prepregs, compounds have a high proportion (50 to 70% by weight) of matrix. During hot compression, the material flows, and fills the entire mould. During this flow, the fibers may be reoriented. The role of the designer will be to find the optimal shape and thickness of the blanks to obtain the desired final characteristics of the part after hardening.

Thus, compounds allow the manufacture of very large parts with possibly complex shapes including ribs and deep-drawn bosses, such as truck and car cabins, etc. Moulding parts with increasingly large surface areas leads to the use of increasingly powerful presses.

Large tonnages were first produced in the field of electrical engineering (figure 3.18), then in the automotive sector (for example, car bumpers, car engine hoods, etc.). The applications of compounds now affect all industrial sectors.



Figure 3.18: Electric metering box for individual houses.

Chapter 4: Composite properties

4.1. Micromechanical homogenization

A composite material is mainly composed of the matrix and reinforcement assembly. Since the reinforcement is responsible for the mechanical strength of the composite, its addition increases and influences the mechanical properties of the reinforced matrix. This is a composite composed of a unidirectional ply (Figure 4.1).

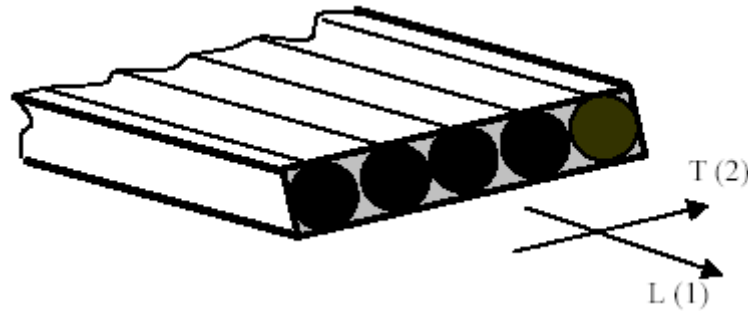


Figure 4.1: Constituents of a composite material

This material is heterogeneous by its nature, its properties depend on those of its components (fibers and matrix). To study the properties of this composite, it is assumed that it is homogeneous and its average properties with respect to the applied external loads are taken into consideration. One of the essential factors in determining the characteristics of a composite material is the relative proportion of the reinforcement and the matrix. This can be expressed in volume fraction or in weight fraction. The weight fraction is easy to measure when developing a composite. The volume fraction is rather used in theoretical models that describe the mechanical behaviour of these materials.

4.2. Rule of mixture (ROM)

4.2.1. Volume fractions

Assuming a volume (v_c) of a composite material of a volume (v_f) of fibers and a volume (v_m) of matrix. The volume fraction of fibers is defined as:

$$V_f = \frac{v_f}{v_c} \quad (4.1)$$

The matrix volume fraction is defined by:

$$V_m = \frac{v_m}{v_c} \quad (4.2)$$

v_f : Volume of fibers

v_m : Volume of matrix

v_c : Volume of composite

Knowing that:

$$v_c = v_m + v_f \quad (4.3)$$

Alors :

$$V_f + V_m = \frac{v_f}{v_c} + \frac{v_m}{v_c} = \frac{v_f + v_m}{v_c} = \frac{v_c}{v_c} = 1 \quad (4.4)$$

Ce qui conduit à :

$$V_m = 1 - V_f \quad (4.5)$$

Et :

$$V_f = 1 - V_m \quad (4.6)$$

4.2.2. Weight fractions

Similarly, the weight fraction can be expressed as a function of the weights (w_c , w_f and w_m) of the composite, the fibres and the matrix respectively. The weight fraction of fibres is defined as follows:

$$W_f = \frac{w_f}{w_c} \quad (4.7)$$

The matrix volume fraction is defined by:

$$W_m = \frac{w_m}{w_c} \quad (4.8)$$

With:

w_f : weight of the fibers

w_m : weight of the matrix

w_c : weight of the composite

Knowing that:

$$w_c = w_m + w_f \quad (4.9)$$

So:

$$W_f + W_m = \frac{w_f}{w_c} + \frac{w_m}{w_c} = \frac{w_f + w_m}{w_c} = \frac{w_c}{w_c} = 1 \quad (4.10)$$

Which leads to:

$$W_m = 1 - W_f \quad (4.11)$$

And

$$W_f = 1 - W_m \quad (4.12)$$

4.2.3. Relationship between volume fractions and weight fractions

The volumetric weight of a material is the weight of this material per unit volume. It is denoted (ρ) and is determined by the relation:

$$\rho = \frac{w}{V} \quad (4.13)$$

The unit of ρ depends on the units chosen for weight (W) and volume (V). In the previous relation:

W: weight

V: volume

ρ : density

According to relation (4.13), the weight of the fibers, the matrix and the composite is given by the following relations:

$$w_f = \rho_f \cdot v_f \quad (4.14)$$

$$w_m = \rho_m \cdot v_m \quad (4.15)$$

$$w_c = \rho_c \cdot v_c \quad (4.16)$$

The total weight of the composite is given by the relation (4.9):

$$w_c = w_m + w_f$$

Or:

$$\rho_c \cdot v_c = \rho_m \cdot v_m + \rho_f \cdot v_f \quad (4.17)$$

If we divide this relationship by the volume of the composite, we obtain the expression of the volumetric weight of the composite as a function of the volume fractions.

$$\rho_c = \rho_m \cdot \frac{v_m}{v_c} + \rho_f \cdot \frac{v_f}{v_c} = \rho_m \cdot V_m + \rho_f \cdot V_f \quad (4.18a)$$

Or:

$$\rho_c = \rho_m \cdot (1 - V_f) + \rho_f \cdot V_f \quad (4.18b)$$

The volume of the composite is given by the relation (4.3):

$$v_c = v_m + v_f \quad (4.19a)$$

Or if we substitute with relations (4.14), (4.15), (4.16), (4.7) and (4.8):

$$\frac{w_c}{\rho_c} = \frac{w_m}{\rho_m} + \frac{w_f}{\rho_f} \quad (4.19b)$$

If we divide this relationship by the weight of the composite, we obtain:

$$\frac{1}{w_c} \left(\frac{w_c}{\rho_c} \right) = \frac{1}{w_c} \left(\frac{w_m}{\rho_m} + \frac{w_f}{\rho_f} \right) = \frac{w_m}{w_c} \left(\frac{1}{\rho_m} \right) + \frac{w_f}{w_c} \left(\frac{1}{\rho_f} \right) = W_m \left(\frac{1}{\rho_m} \right) + W_f \left(\frac{1}{\rho_f} \right) \quad (4.20)$$

The final expression of the volumetric weight of the composite as a function of the weight fractions is given by the following relation:

$$\frac{1}{\rho_c} = \left(\frac{W_m}{\rho_m} + \frac{W_f}{\rho_f} \right) \quad (4.21)$$

So:

$$\rho_c = \frac{1}{\left(\frac{W_m}{\rho_m} + \frac{W_f}{\rho_f} \right)} \quad (4.22)$$

We can thus establish a relationship between weight fractions and volume fractions based on the following definitions:

The mass of the fibers is given by the relationship (4.7):

$$W_f = \frac{w_f}{w_c}$$

So:

$$W_f = \frac{w_f}{w_c} = \frac{\rho_f \cdot v_f}{\rho_c \cdot v_c} = \frac{\rho_f}{\rho_c} V_f \quad (4.23b)$$

Finally:

$$W_f = \frac{\rho_f}{\rho_c} V_f \quad (4.23c)$$

In the same way we end up with:

$$W_m = \frac{\rho_m}{\rho_c} V_m \quad (4.24)$$

If we replace the volumetric weight expressions (4.18) and (4.22) respectively in the relations (4.23) and (4.24) we obtain:

$$W_f = \left(\frac{\rho_f}{\rho_m \cdot (1 - V_f) + \rho_f \cdot V_f} \right) V_f \quad (4.25)$$

And:

$$W_m = \left(\frac{\frac{\rho_m}{1}}{\left(\frac{W_m}{\rho_m} + \frac{W_f}{\rho_f} \right)} \right) V_m \quad (4.26)$$

4.3. Determination of the elastic properties of a unidirectional ply

A unidirectional composite consists of parallel fibers arranged in a matrix. This type of material constitutes the basic configuration of fiber composite materials, hence the importance of its study.

4.3.1. Longitudinal elasticity modulus

Simplified and convenient expressions of the moduli can be obtained by making a very simplified approach to the mechanical behaviour of the elementary cell of the unidirectional composite material. The longitudinal Young's modulus is determined in a longitudinal tensile test (Figure 4.2). The simplifying hypothesis is to assume an identical deformation in the fiber and in the matrix.

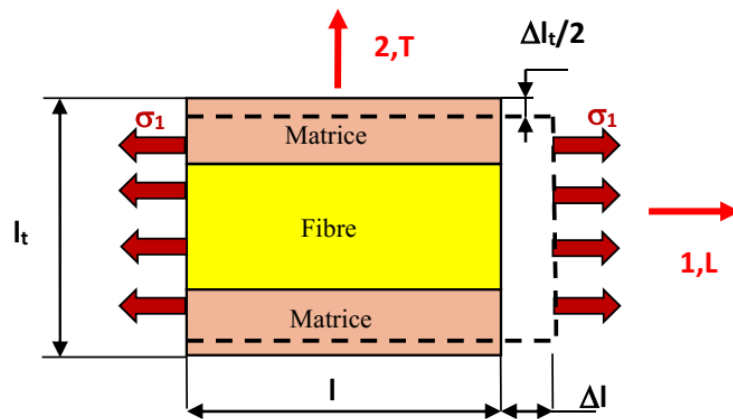


Figure 4.2: Elementary cell loaded longitudinally

Under the action of a tensile force the composite elongates by an amount (ΔL). The deformation in this case is defined by the relation:

$$\epsilon_L = \frac{\Delta L}{L} \quad (4.28)$$

This deformation is identical in the fiber and in the matrix:

$$\epsilon_f = \epsilon_m = \epsilon_c = \epsilon_L \quad (4.29)$$

We know that for an elastic material and if we apply Hooke's law:

$$\sigma_f = E_f \epsilon_f = E_f \epsilon_L \quad (4.30)$$

And:

$$\sigma_m = E_m \epsilon_m = E_m \epsilon_L \quad (4.31)$$

The force F in the fibers and in the matrix can be expressed by:

$$F_f = \sigma_f S_f = E_f \varepsilon_L S_f \quad (4.32)$$

And:

$$F_m = \sigma_m S_m = E_m \varepsilon_L S_m \quad (4.33)$$

S_f and S_m represent the cross-sections of the fibers and the matrix, respectively. The total force applied in the longitudinal direction is:

$$F_L = F_1 = F_f + F_m \quad (4.34)$$

Or:

$$F_L = F_1 = \sigma_f S_f + \sigma_m S_m \quad (4.35)$$

If (S) is the cross-section of the composite, we have:

$$\sigma_L = \sigma_1 = \frac{F_1}{S} \quad (4.36)$$

If we substitute relation (4.35) into (4.36) we obtain:

$$\sigma_L = \sigma_1 = \frac{F_1}{S} = \frac{\sigma_f S_f + \sigma_m S_m}{S} = \sigma_f \frac{S_f}{S} + \sigma_m \frac{S_m}{S} \quad (4.37)$$

Let's consider the cross section of the composite as shown in Figure 4.3:

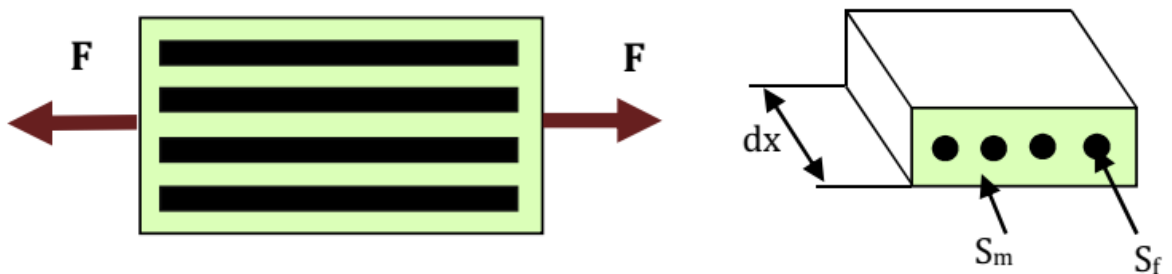


Figure 4.3: Cross section of composite

Knowing that:

$$V_f = \frac{v_f}{v_c} = \frac{S_f \cdot dx}{S_c \cdot dx} = \frac{S_f}{S_c} \quad (4.38)$$

And:

$$V_m = \frac{v_m}{v_c} = \frac{S_m \cdot dx}{S_c \cdot dx} = \frac{S_m}{S_c} \quad (4.39)$$

Relation (4.37) becomes:

$$\sigma_L = \sigma_1 = \sigma_f V_f + \sigma_m V_m \quad (4.40)$$

Or even:

$$\sigma_L = \sigma_1 = \sigma_f V_f + \sigma_m (1 - V_f) \quad (4.41)$$

This average stress is linked to the deformation of the cell by the longitudinal modulus of elasticity in the longitudinal direction:

$$\sigma_L = \sigma_1 = E_L \varepsilon_L \quad (4.42)$$

The combination of relations (4.41) and (4.42) leads to the expression of the longitudinal Young's modulus:

$$E_L = \frac{\sigma_L}{\varepsilon_L} = \frac{\sigma_f V_f + \sigma_m (1 - V_f)}{\varepsilon_L} \quad (4.43)$$

Knowing that:

$$\varepsilon_L = \varepsilon_f = \varepsilon_m \quad (4.44)$$

Then:

$$E_L = \frac{\sigma_f V_f + \sigma_m (1 - V_f)}{\varepsilon_L} = \frac{\sigma_f}{\varepsilon_f} V_f + \frac{\sigma_m}{\varepsilon_m} (1 - V_f) = E_f V_f + E_m (1 - V_f) \quad (4.45)$$

$$E_L = E_f V_f + E_m (1 - V_f) \quad (4.46)$$

This expression is known as the law of mixtures which allows the calculation of Young's modulus in the direction of the fibers.

4.3.2. Transverse elastic modulus

The transverse elastic modulus or transverse Young's modulus can be determined by a transverse tensile test in which the composite is stressed in tension relative to a direction perpendicular to the fibres (Figure 4.4). We consider a representative elementary cell (REC) made up of fiber of height (h_f) surrounded by matrix of height (h_m) stressed in traction (σ_2 or σ_T) with respect to the transverse direction (2 or T).

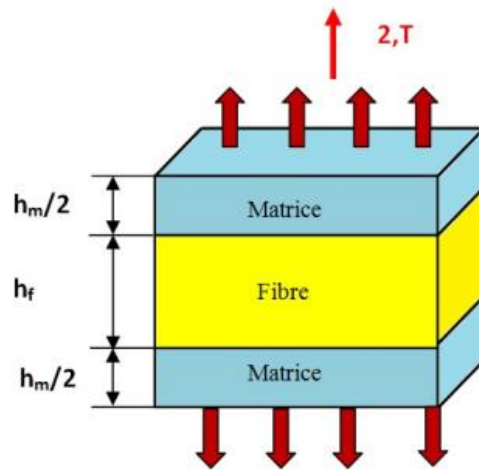


Figure 4.4: Elementary cell loaded transversely

Let's express the volume fractions of the fibers and the matrix as a function of the heights of the layers (matrix + fiber).

$$V_f = \frac{v_f}{v_c} = \frac{S_f \cdot e}{S_c \cdot e} = \frac{h_f \cdot L \cdot e}{(h_f + h_m) \cdot L \cdot e} = \frac{h_f}{(h_f + h_m)} \quad (4.47)$$

With:

L: length of the REC

e: thickness of the REC

Similarly:

$$V_m = 1 - V_f = \frac{h_m}{(h_f + h_m)} \quad (4.48)$$

The force F_2 imposed in the transverse direction is completely transmitted in the fiber and in the matrix leading to equal stresses.

$$\sigma_m = \sigma_f = \sigma_2 \quad (4.49)$$

The resulting deformations in the fiber and in the matrix in the transverse direction are given by the relations:

$$\varepsilon_f = \frac{\sigma_2}{E_f} \quad (4.50)$$

And:

$$\varepsilon_m = \frac{\sigma_2}{E_m} \quad (4.51)$$

The transverse elongation in the representative elementary cell (REC), resulting from the elongations in the fiber and in the matrix is given by the following relation:

$$\Delta L_2 = \varepsilon_f h_f + \varepsilon_m h_m \quad (4.52)$$

And the transverse deformation:

$$\varepsilon_2 = \frac{\Delta L_2}{h_f + h_m} = \varepsilon_f \frac{h_f}{(h_f + h_m)} + \varepsilon_m \frac{h_m}{(h_f + h_m)} \quad (4.53)$$

If we substitute (4.47) and (4.48) in (4.53) we arrive at:

$$\varepsilon_2 = \varepsilon_f V_f + \varepsilon_m V_m = \varepsilon_f V_f + \varepsilon_m (1 - V_f) \quad (4.54a)$$

The combination of relations (4.50), (4.51) and (4.54a) leads to:

$$\varepsilon_2 = \frac{\sigma_2}{E_T} = \varepsilon_f V_f + \varepsilon_m (1 - V_f) = \frac{\sigma_2}{E_f} V_f + \frac{\sigma_2}{E_m} (1 - V_f) \quad (4.54b)$$

In this relation (E_T) represents the transverse Young's modulus. This leads after simplification to:

$$\frac{1}{E_T} = \frac{1}{E_f} V_f + \frac{1}{E_m} (1 - V_f) \quad (4.55a)$$

Eventually:

$$\frac{1}{E_T} = \frac{V_f}{E_f} + \frac{(1 - V_f)}{E_m} \quad (4.55b)$$

This expression is known as the inverse law of mixtures and allows the calculation of the transverse Young's modulus in the direction perpendicular to that of the fibers.

The relation (4.55) can be written in another dimensionless form:

$$\frac{E_m}{E_T} = \frac{E_m V_f}{E_f} + \frac{E_m (1 - V_f)}{E_m} = \frac{E_m}{E_f} V_f + (1 - V_f) = 1 + V_f \left(\frac{E_m}{E_f} - 1 \right) \quad (4.56a)$$

Then:

$$\frac{E_T}{E_m} = \frac{1}{1 + V_f \left(\frac{E_m}{E_f} - 1 \right)} \quad (4.56b)$$

Likewise:

$$\frac{E_f}{E_T} = \frac{E_f V_f}{E_f} + \frac{E_f (1 - V_f)}{E_m} = \frac{E_f}{E_f} V_f + (1 - V_f) \frac{E_f}{E_m} = V_f + (1 - V_f) \frac{E_f}{E_m} = 1 + V_f \left(1 - \frac{E_f}{E_m} \right)$$

Then:

$$\frac{E_T}{E_f} = \frac{1}{1 + V_f \left(1 - \frac{E_f}{E_m} \right)} \quad (4.57)$$

4.3.3. Shear modulus

The shear modulus G_{LT} is determined from a shear test schematized in Figure 4.5. We consider a representative elementary cell (REC) of a unidirectional composite, consisting of fiber and matrix. The shear stresses in the fiber and in the matrix are equal (iso stresses) because the stress τ is assumed to be imposed on the entire cell composed of fiber and matrix.

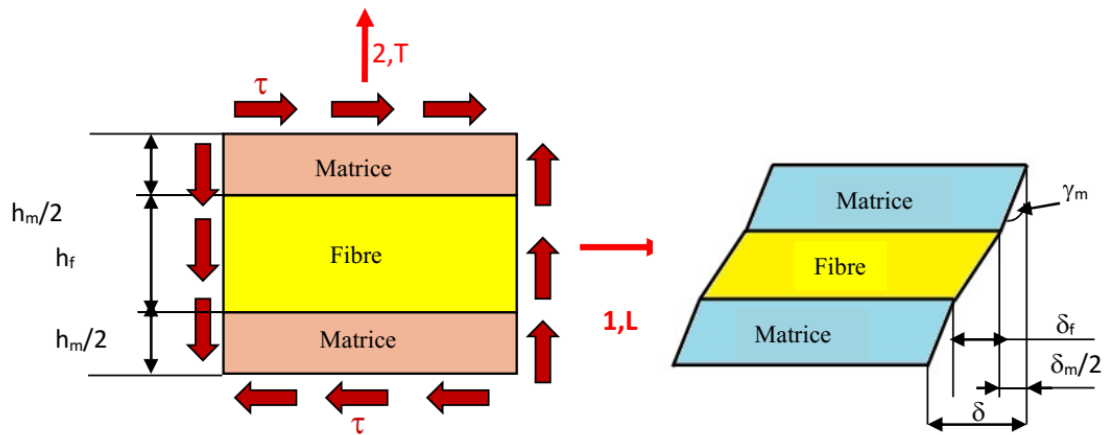


Figure 4.5: Elementary cell subjected to shearing

The shear deformations (distortions) in the fiber and in the matrix are expressed by the following relations:

$$\gamma_f = \frac{\tau}{G_f} \quad (4.58)$$

And:

$$\gamma_m = \frac{\tau}{G_m} \quad (4.59)$$

The deformations generated in the fiber and in the matrix are given by the following relations:

$$\delta_f = h_f \cdot \gamma_f \quad (4.60)$$

And:

$$\delta_m = h_m \cdot \gamma_m \quad (4.61)$$

The total strain in the cell (REC) is:

$$\delta = \delta_f + \delta_m \quad (4.62)$$

Or either:

$$\delta = h_f \cdot \gamma_f + h_m \cdot \gamma_m \quad (4.63)$$

The overall shear angle in the cell is:

$$\gamma = \frac{\delta}{h_f + h_m} \quad (4.64)$$

If we replace (4.63) in (4.64) we obtain:

$$\gamma = \frac{h_f \cdot \gamma_f + h_m \cdot \gamma_m}{h_f + h_m} = \left(\frac{h_f}{h_f + h_m} \right) \gamma_f + \left(\frac{h_m}{h_f + h_m} \right) \gamma_m \quad (4.65a)$$

Knowing that :

$$V_f = \frac{h_f}{(h_f + h_m)} \quad (4.65b)$$

And:

$$V_m = 1 - V_f = \frac{h_m}{(h_f + h_m)} \quad (4.65c)$$

Then:

$$\gamma = \gamma_f V_f + \gamma_m (1 - V_f) \quad (4.66)$$

This angle is related to the shear stress by the shear modulus G_{LT} by the relation:

$$\gamma = \frac{\tau}{G_{LT}} \quad (4.67)$$

By combining the relations (4.66) and (4.67), we end up with:

$$\gamma = \gamma_f V_f + \gamma_m (1 - V_f) = \frac{\tau}{G_{LT}} = \frac{\tau}{G_f} V_f + \frac{\tau}{G_m} (1 - V_f) \quad (4.68)$$

Which leads to:

$$\frac{1}{G_{LT}} = \frac{V_f}{G_f} + \frac{1 - V_f}{G_m} \quad (4.69)$$

There is a certain analogy between this relation (4.69) and the relation (4.55) which expresses the transverse shear modulus E_T .

4.3.4. Poisson's ratio

Poisson's ratio characterizes the ability of a material to compress (Figure 4.6). It reflects the ratio of the relative transverse elongation ε_T and longitudinal ε_L :

$$\varepsilon_T = -\nu \cdot \varepsilon_L \quad (4.70)$$

Poisson's ratio is an essential parameter in the characterization of the elastic properties of a material. It is inaccessible by direct measurement; however, it is determined by experimental measurement using the following relationship:

$$\nu = \frac{\Delta l}{\Delta L} = \frac{\frac{(l_0 - l)}{l_0}}{\frac{(L - L_0)}{L_0}} = -\frac{\frac{(l - l_0)}{l_0}}{\frac{(L - L_0)}{L_0}} = -\frac{\varepsilon_T}{\varepsilon_L} \quad (4.71)$$

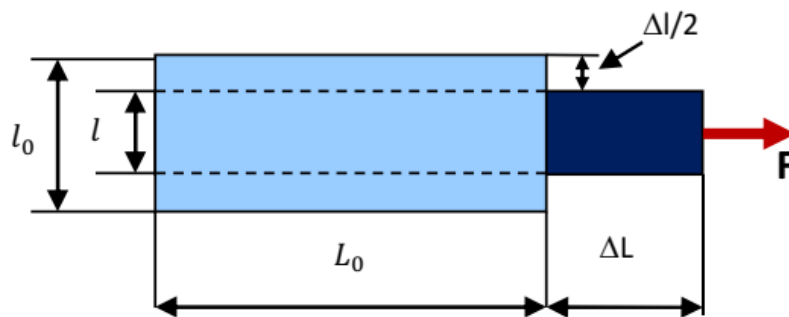


Figure 4.6: Poisson's ratio

To determine the Poisson's ratio for the case of a composite material, let's consider a (REC) of this material (Figure 4.7)

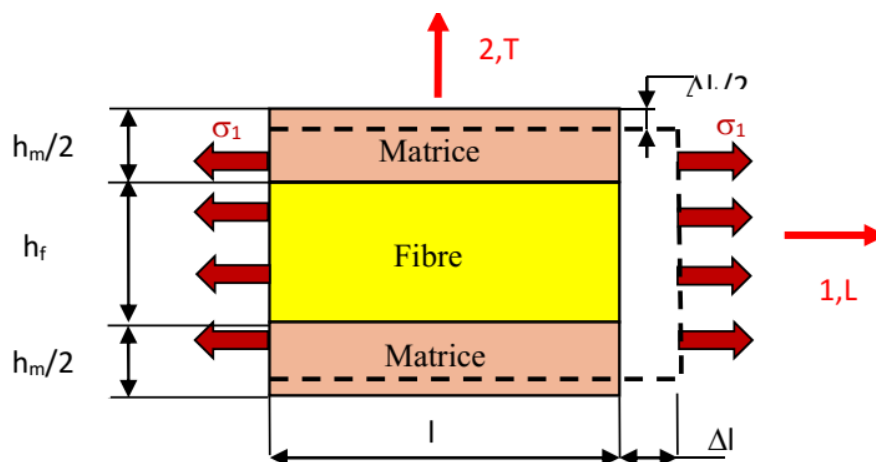


Figure 4.7: Representative elementary cell loaded in the fiber direction

For a tensile test the relative elongation of the composite in the transverse direction is defined by:

$$\varepsilon_2 = \varepsilon_T = \frac{\Delta l_T}{l_T} \quad (4.72)$$

The relative elongation of the composite in the longitudinal direction is defined by:

$$\varepsilon_1 = \varepsilon_L = \varepsilon_f = \varepsilon_m \quad (4.73)$$

The deformations in the matrix and in the fibers are given by the relations:

$$\varepsilon_{2,m} = -\nu_m \cdot \varepsilon_1 \quad (4.74)$$

And

$$\varepsilon_{2,f} = -\nu_f \cdot \varepsilon_1 \quad (4.75)$$

On the other hand, the relative transverse elongation of the REC is:

$$\Delta h = \Delta h_f + \Delta h_m \quad (4.76)$$

Knowing that in general the relative deformation is defined by the ratio of the elongation to the initial length:

$$\varepsilon = \frac{\Delta L}{L} \quad (4.77)$$

So :

$$\varepsilon_{2,f} = \frac{\Delta h_f}{h_f} \quad (4.78)$$

And

$$\varepsilon_{2,m} = \frac{\Delta h_m}{h_m} \quad (4.79)$$

Thus, the relation (4.76) becomes:

$$\Delta h = \varepsilon_{2,f} \cdot h_f + \varepsilon_{2,m} \cdot h_m \quad (4.80)$$

If we substitute (4.74) and (4.75) into (4.80), we obtain:

$$\Delta h = -\nu_f \cdot \varepsilon_1 \cdot h_f - \nu_m \cdot \varepsilon_1 \cdot h_m \quad (4.81)$$

D'autre part la déformation totale de la REC dans le sens transversal est donnée par la relation :

$$\varepsilon_2 = \frac{\Delta h}{h_f + h_m} \quad (4.82)$$

Or

$$\varepsilon_2 = \frac{-\nu_f \cdot \varepsilon_1 \cdot h_f - \nu_m \cdot \varepsilon_1 \cdot h_m}{h_f + h_m} = \left(-\nu_f \frac{h_f}{h_f + h_m} - \nu_m \frac{h_m}{h_f + h_m} \right) \varepsilon_1 \quad (4.83)$$

With

$$V_f = \frac{h_f}{h_f + h_m} \quad (4.84)$$

And

$$V_m = (1 - V_f) = \frac{h_m}{h_f + h_m} \quad (4.85)$$

Then

$$\varepsilon_2 = \left(-\nu_f V_f - \nu_m (1 - V_f) \right) \varepsilon_1 \quad (4.86)$$

Finally, we arrive at the expression of the transverse Poisson's ratio ν_{LT} :

$$\begin{aligned} \nu_{LT} &= -\frac{\varepsilon_2}{\varepsilon_1} = -\frac{\varepsilon_T}{\varepsilon_L} = \nu_f V_f + \nu_m (1 - V_f) \\ \nu_{LT} &= \nu_f V_f + \nu_m (1 - V_f) \end{aligned} \quad (4.87)$$

4.3.5. Analytical methods for determining the elastic moduli of the ply

The comparison between the values of the moduli determined experimentally on unidirectionals and evaluated by the rule of mixtures showed that only formulas (4.46) and (4.69) give values close to those obtained experimentally. In addition, this method does not allow the evaluation of the shear modulus G_{TT} . A more precise approach consists in developing models that take into account the geometry of the fibers and their arrangement inside the matrix. By considering a cylindrical arrangement (elementary cell consisting of a fiber surrounded by a resin cylinder), the resolution of the corresponding elasticity problem allows to arrive at exact analytical solutions representing the behaviour of the composite

consisting of a uniform distribution of this cell. The independent moduli obtained by this approach are:

4.3.5.1. Longitudinal elastic modulus

$$E_L = E_f V_f + E_m (1 + V_f) + \frac{4V_f(1 - V_f)(v_f - v_m)^2}{\frac{V_f}{K_m} + \frac{1}{G_m} + \frac{1 - V_f}{K_f}} \approx E_f V_f + E_m (1 - V_f) \quad (4.88)$$

4.3.5.2. Poisson's ratio ν_{LT}

$$\nu_{LT} = \nu_f V_f + \nu_m (1 - V_f) + \frac{V_f(1 - V_f)(v_f - v_m) \left(\frac{1}{K_m} - \frac{1}{K_f} \right)}{\frac{V_f}{K_m} + \frac{1}{G_m} + \frac{1 - V_f}{K_f}} \quad (4.89)$$

4.3.5.3. Shear modulus G_{LT}

$$G_{LT} = G_m \frac{G_f(1 + V_f) + G_m(1 - V_f)}{G_f(1 - V_f) + G_m(1 + V_f)} \quad (4.90)$$

4.3.5.4. Compressibility modulus (matrix)

$$k_m = \frac{E_m}{3(1 - V_f) + G_m(1 + V_f)} \quad (4.91)$$

4.3.5.5. Lateral compression modulus K_L

$$K_L = K_m + \frac{V_f}{K_f - K_m + \frac{(G_f - G_m)}{3} + \frac{1 - V_f}{K_m + \frac{4}{3}G_m}} \quad (4.92)$$

4.3.5.6. Transverse shear modulus $G_{TT'}$

$$G_{TT'} = G_m \left[1 + \frac{V_f}{\frac{G_m}{G_f - G_m} + \frac{K_m + \frac{7}{3}G_m}{2K_m + \frac{8}{3}G_m} (1 - V_f)} \right] \quad (4.93)$$

4.3.5.7. Transverse elastic modulus

The transverse Young's modulus is calculated from the previous moduli using the elasticity relation:

$$E_T = \frac{2}{\frac{1}{2K_L} + \frac{1}{2G_{TT'}} + 2\frac{\nu_{LT}^2}{E_L}} \quad (4.94)$$

Chapter 5: Behaviour of composites to internal and external actions

5.1. Generalized Hooke's Law

The relationship that expresses the linearity between stresses and strains is written as follows:

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} \quad (5.1)$$

Where: C_{ijkl} is a fourth-order tensor with 81 elastic constants.

The symmetry of the stress tensor gives: $C_{ijkl} = C_{jikl}$ (since $\sigma_{ij} = \sigma_{ji}$). In the same way, the symmetry of the strain tensor gives: $C_{ijkl} = C_{ijlk}$ (since $\varepsilon_{kl} = \varepsilon_{lk}$).

Thus, the number of independent constants is reduced from 81 to 36.

Because of this reduced number of constants, Hooke's law can be written in contracted notation as follows:

$$\sigma_i = C_{ij} \varepsilon_j \quad (i, j = 1, 2, \dots, 6) \quad (5.2)$$

Where the contracted notation of stresses and strains is defined by the following equivalences:

$$\begin{array}{ll} \sigma_1 \equiv \sigma_{11} & \varepsilon_1 \equiv \varepsilon_{11} \\ \sigma_2 \equiv \sigma_{22} & \varepsilon_2 \equiv \varepsilon_{22} \\ \sigma_3 \equiv \sigma_{33} & \varepsilon_3 \equiv \varepsilon_{33} \\ \sigma_4 \equiv \sigma_{23} & \varepsilon_4 \equiv 2\varepsilon_{23} \\ \sigma_5 \equiv \sigma_{31} & \varepsilon_5 \equiv 2\varepsilon_{31} \\ \sigma_6 \equiv \sigma_{12} & \varepsilon_6 \equiv 2\varepsilon_{12} \end{array} \quad (5.3)$$

It is noted here that in the contracted notation, the terms of the shear strain are defined as above:

For example: $\varepsilon_4 = 2\varepsilon_{23} = \gamma_{23}$

C_{ij} are referred to by a variety of names: Elastic constants, elastic modulus, and stiffness coefficients.

For equation (5.2) to have a solution, it is necessary that: $|C_{ij}| \neq 0$.

The number of independent elastic constants can be further reduced if there is a density function of the strain energy "W" such that:

$$W = \frac{1}{2} C_{ij} \varepsilon_i \varepsilon_j \quad (5.4)$$

With the property:

$$\sigma_i = \frac{\partial W}{\partial \varepsilon_i} \quad (5.5)$$

If the quadratic form (5.4) exists in such a way that property (5.5) is true, then " C_{ij} " is symmetric and the number of constants is reduced accordingly i.e.:

$$C_{ij} = C_{ji} \quad (5.6)$$

$$[C_{ij}] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \quad (5.7)$$

This matrix is symmetric with 21 independent constants.

The existence of the function W is based on the first and second laws of thermodynamics.

The experimental results are in agreement with this theory for an elastic response of the material.

The inverted form of Hooke's law is written as follows:

$$\varepsilon_i = S_{ij} \sigma_j \quad (5.8)$$

We note here that: since the stiffness matrix is symmetrical, so is the flexibility matrix.

A material with 21 independent constants is called "**Anisotropic**"

The maximum number of independent constants for an anisotropic material is limited by the symmetry of the stress tensor and the strains and the assumption of the existence of the strain energy function.

5.2 Material symmetry

5.2.1. Monoclinic Materials

Depending on the symmetry of the material, the number of independent constants decreases.

For example, the plane (X_1 - X_2) of the equation $X_3 = 0$ is a plane of symmetry, all constants associated with the positive direction of the X_3 axis must be identical to those associated with the negative direction of the same X'_3 axis (see Figure 5.1)

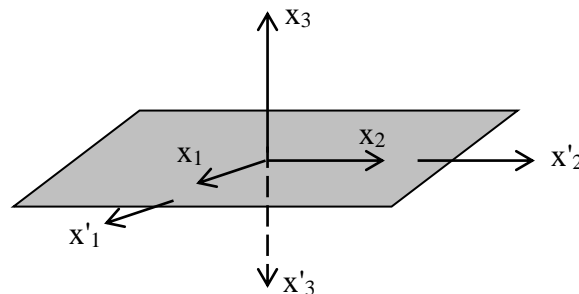


Figure 5.1: Symmetry with respect to the plane x_1 - x_2

The application of this symmetry can be deduced by considering Hooke's law in each coordinate system:

X_1 - X_2 - X_3 reference point:

$$\sigma_i = C_{ij} \varepsilon_j \quad (5.9)$$

X'_1 - X'_2 - X'_3 reference point:

$$\sigma'_i = C'_{ij} \varepsilon'_j$$

Given the symmetry with respect to the X_1 - X_2 plane, we will have:

$$\begin{aligned} \sigma'_1 &= \sigma_1 & \sigma'_4 &= -\sigma_4 \\ \sigma'_2 &= \sigma_2 & \sigma'_5 &= -\sigma_5 \\ \sigma'_3 &= \sigma_3 & \sigma'_6 &= \sigma_6 \end{aligned} \quad (5.11)$$

$$\begin{aligned} \varepsilon'_1 &= \varepsilon_1 & \varepsilon'_4 &= -\varepsilon_4 \\ \varepsilon'_2 &= \varepsilon_2 & \varepsilon'_5 &= -\varepsilon_5 \\ \varepsilon'_3 &= \varepsilon_3 & \varepsilon'_6 &= \varepsilon_6 \end{aligned} \quad (5.12)$$

Expressing the first equality $\sigma'_1 = \sigma_1$ in equation (5.11) as a function of deformations and stiffnesses using equations (5.9) and (5.10), we will have:

$$C'_{11} \varepsilon'_1 + C'_{12} \varepsilon'_2 + \dots + C'_{16} \varepsilon'_6 = C_{11} \varepsilon_1 + C_{12} \varepsilon_2 + \dots + C_{16} \varepsilon_6 \quad (5.13)$$

Replacing the terms ε'_i according to ε_i equation (5.12) and comparing similar terms in equation (5.13), we get:

$$C_{14} = C_{15} = 0 \quad (5.14)$$

Similarly, the remaining equations in (5.11) lead us to the following results:

$$C_{24} = C_{25} = C_{34} = C_{35} = C_{46} = C_{56} = 0 \quad (5.15)$$

Thus, the material symmetry with respect to the X_1 - X_2 plane, reduces the stiffness matrix to:

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix} \quad (5.16)$$

As shown in equation (5.16), a material with a material plane of symmetry has 13 independent elastic constants.

The effective properties of a unidirectional composite with off-axis oriented fibers can be modelled as those of a homogeneous anisotropic material. As shown in Figure 5.2, two Cartesian coordinate systems are identified: the system $(X_1-X_2-X_3)$ and the system $(X-Y-Z)$. The X_1 direction is that of the fibers, with the X_2-X_3 directions perpendicular to the latter.

The coordinate system $(X-Y-Z)$ is obtained by rotating with respect to the X_3 axis. The material has symmetry with respect to the $X-Y$ plane and not with respect to $X-Z$ or $Y-Z$.

Such a composite material with this plane of symmetry is called **a monoclinic material**.

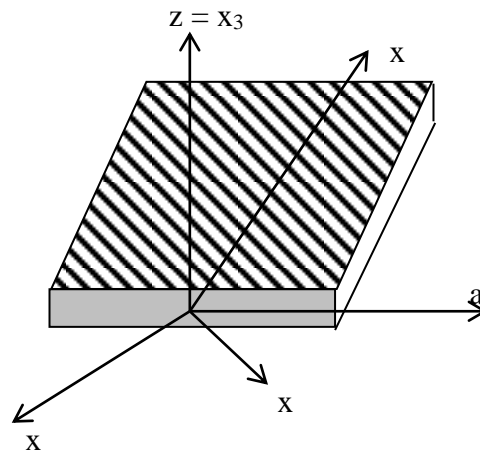


Figure 5.2: Used coordinate system

5.2.2. Orthotropic materials

A medium is said to be orthotropic for a given property, if this property is invariant by change of direction obtained by symmetry relative to the two orthogonal planes (X_1-X_2) and (X_2-X_3) (Figure 5.3). Otherwise, if the characteristics of a specimen cut along symmetrical axes with respect to a particular plane $P1(X_1-X_2)$ and then with respect to another plane $P2(X_2-X_3)$ are identical, then the material is orthotropic with an orthotropic plane $P1$, $P2$ and obviously $P3$.

The intersections of these three planes of symmetry define the main axes of orthotropy. It is a model that schematizes well the composition of unidirectional composites and woods, but also of rolled metal products.

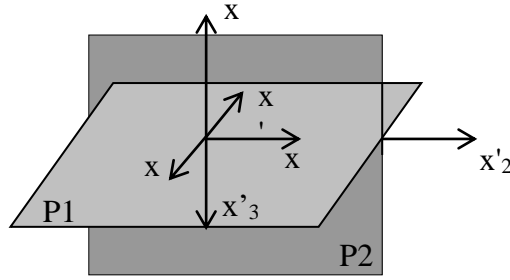


Figure 5.3: Symmetry with respect to the planes x_1-x_2 and x_2-x_3

The stresses $\sigma_1, \sigma_2, \sigma_3$ et σ_5 must be equal to the stresses corresponding to the coordinate system $X'_1 X'_2 X'_3$.

Following the same reasoning as that of paragraph 5.1, we will have:

$$C_{16} = C_{26} = C_{36} = C_{46} = 0 \quad (5.17)$$

The stiffness matrix of an orthotropic material has nine (09) independent elastic constants:

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \quad (5.18)$$

5.2.3. Transversely isotropic materials

An elastic medium has a transverse isotropy if its elasticity characteristics are identical for all symmetrical direction couples with respect to an axis. If the transverse isotropy axis is located by X_3 , the material is isotropic in all planes normal to X_3 .

A laminated composite obtained by coupling layers formed of unidirectional fibers arranged in all directions can be considered transverse isotropic.

For these materials, the non-zero constants are nine in number, of which five (05) are independent. This is an important configuration because it is often used to describe the elastic constants of a unidirectional fiber or material.

In both cases, the isotropy plane is normal to the direction of the fibers. The change from nine to five non-zero constants is done using the following relations:

$$C_{22} = C_{33}, C_{12} = C_{13}, C_{55} = C_{66}, C_{44} = \frac{C_{22} - C_{23}}{2} \quad (5.19)$$

The stiffness matrix of a transversely isotropic material is as follows:

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{C_{22} - C_{23}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \quad (5.20)$$

5.2.4. Isotropic materials

If the material has an infinite number of planes of symmetry, it is said to be **isotropic**, in which case the number of independent constants is only two (02):

$$C_{11} = C_{22}, C_{12} = C_{23}, C_{66} = \frac{C_{22} - C_{23}}{2} = \frac{C_{11} - C_{22}}{2} \quad (5.21)$$

The stiffness matrix of such a material is given by:

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{C_{11} - C_{12}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{C_{11} - C_{12}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11} - C_{12}}{2} \end{bmatrix} \quad (5.22)$$

5.3. Engineering constants

The simplest form of the constituent equations is obtained by writing them in terms of the elements of the stiffness C_{ij} or flexibility S_{ij} matrix. However, in general, coefficients are not measured directly in the laboratory. The constants that are measured in the laboratory are called "engineer's constants".

In order to establish a relationship between the engineering constants and the coefficients of the stiffness matrix, we consider an orthotropic material.

The axes are aligned parallel and perpendicular to the direction of the fibers (Figure 5.4). The deformation-stress equations via the engineering constants are:

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - \frac{\nu_{21}}{E_2} \sigma_2 - \frac{\nu_{31}}{E_3} \sigma_3 \quad (5.23)$$

$$\varepsilon_2 = -\frac{\nu_{12}}{E_1} \sigma_1 + \frac{\sigma_2}{E_2} - \frac{\nu_{32}}{E_3} \sigma_3 \quad (5.24)$$

$$\varepsilon_3 = -\frac{\nu_{13}}{E_1} \sigma_1 - \frac{\nu_{23}}{E_2} \sigma_2 + \frac{\sigma_3}{E_3} \quad (5.25)$$

$$\gamma_{23} = \frac{\tau_{23}}{G_{23}} \quad (5.26)$$

$$\gamma_{13} = \frac{\tau_{13}}{G_{13}} \quad (5.27)$$

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}} \quad (5.28)$$

E_1 : Longitudinal Young's modulus (direction of the fibres);

E_2, E_3 : Transversal Young's modulus;

ν_{ij} : Poisson's ratio (the first subscript "i" corresponds to the direction of the applied stress, the second subscript "j" corresponds to the direction of the associated lateral deformation) thus, for $\sigma_i \neq 0, i \neq j$ et $i, j = 1, 2, 3$

$$\nu_{ij} = -\frac{\varepsilon_j}{\varepsilon_i} \quad (5.29)$$

It is important to note that in the general case $\nu_{ij} \neq \nu_{ji}$.

The shear modulus G_{ij} is defined by a shear stress load in the plane i - j . Thus, for a unidirectional fibrous composite, G_{12} and G_{13} are axial shear moduli in the two orthogonal planes that contain the fibers, and G_{23} is the transverse shear modulus.

The shear moduli corresponding to the shear stiffness of the associated plane and consequently: $G_{ij} = G_{ji}$.

The constitutive equations (5.8) for an orthotropic material can be written

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} \quad (5.30)$$

Where, we used: $\gamma_{ij} = 2\varepsilon_{ij}$

Equalizing the equivalent terms in (5.23 to 5.28) with those in equation (5.30), we will have:

$$\begin{aligned} S_{11} &= \frac{1}{E_1}, & S_{12} &= -\frac{\nu_{21}}{E_2}, & S_{13} &= -\frac{\nu_{31}}{E_3} \\ S_{12} &= -\frac{\nu_{12}}{E_1}, & S_{22} &= \frac{1}{E_2}, & S_{23} &= -\frac{\nu_{23}}{E_3} \\ S_{31} &= -\frac{\nu_{13}}{E_1}, & S_{32} &= -\frac{\nu_{23}}{E_2}, & S_{33} &= \frac{1}{E_3} \\ S_{44} &= \frac{1}{G_{23}}, & S_{55} &= \frac{1}{G_{13}}, & S_{66} &= \frac{1}{G_{12}} \end{aligned} \quad (5.31)$$

Using the symmetry of the elements of the flexibility matrix ($S_{ij}=S_{ji}$):

$$\frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1}, \quad \frac{\nu_{31}}{E_3} = \frac{\nu_{13}}{E_1}, \quad \frac{\nu_{32}}{E_3} = \frac{\nu_{23}}{E_2} \quad (5.32)$$

Often, relations (5.32) are written as follows:

$$E_1 \nu_{21} = E_2 \nu_{12}, \quad E_1 \nu_{31} = E_3 \nu_{13}, \quad E_2 \nu_{32} = E_3 \nu_{23} \quad (5.33)$$

As noted above, in general $\nu_{ij} \neq \nu_{ji}$. For example, from equation (5.33) we will have:

$$\nu_{21} = \frac{E_2}{E_1} \nu_{12}; \quad \nu_{32} = \frac{E_3}{E_2} \nu_{23} \quad (5.34)$$

So for a transversely isotropic material with $E_1 \gg E_2$ and $E_2 = E_3$, ν_{21} is very small in front of ν_{12} ($\nu_{21} \ll \nu_{12}$) but $\nu_{23} = \nu_{32}$.

Let us substitute equations (5.31) in (5.30):

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_3} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} \quad (5.35)$$

Inverting equation (5.35), we will have:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1-\nu_{23}\nu_{32}}{E_2 E_3 \Delta} & \frac{\nu_{21} + \nu_{23}\nu_{31}}{E_2 E_3 \Delta} & \frac{\nu_{31} + \nu_{21}\nu_{32}}{E_2 E_3 \Delta} & 0 & 0 & 0 \\ \frac{\nu_{21} + \nu_{23}\nu_{31}}{E_2 E_3 \Delta} & \frac{1-\nu_{13}\nu_{31}}{E_2 E_3 \Delta} & \frac{\nu_{32} + \nu_{12}\nu_{31}}{E_2 E_3 \Delta} & 0 & 0 & 0 \\ \frac{\nu_{31} + \nu_{21}\nu_{32}}{E_2 E_3 \Delta} & \frac{\nu_{32} + \nu_{12}\nu_{31}}{E_2 E_3 \Delta} & \frac{1-\nu_{12}\nu_{21}}{E_2 E_3 \Delta} & 0 & 0 & 0 \\ \frac{0}{E_2 E_3 \Delta} & \frac{0}{E_2 E_3 \Delta} & \frac{0}{E_2 E_3 \Delta} & G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} \quad (5.36)$$

$$\text{With: } \Delta = \frac{(1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - 2\nu_{21}\nu_{32}\nu_{13})}{(E_1 E_2 E_3)}$$

Δ Is the determinant of the flexibility matrix (equation (5.35)).

For transversely isotropic materials with:

$$E_2 = E_3, \quad \sigma_{12} = \sigma_{13}, \quad G_{12} = G_{13} \text{ and } G_{23} = \frac{E_2}{2(1+\nu_{23})} \text{ and using the relationships}$$

$E - \nu$ given by equations (5.32) and (5.35).

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{12}}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu_{23})}{E_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{12}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} \quad (5.37)$$

5.4. Transformation of coordinates

Stresses and strains are second-order tensors, but the stiffness matrix and the flexibility matrix are fourth-order tensors. Consequently, these quantities obey the laws of transformation of tensors.

In the following, we will use matrix notation to develop the transformed stiffness and flexibility matrices.

The vector of σ and ε in the principal coordinate system (1, 2, 3) is denoted $\{\}_{1}$ and $\{\}_{x}$ in the coordinate system (X, Y, Z).

Rotation is considered through an angle θ about a common "Z" axis (3) as shown in Figure 5.4:

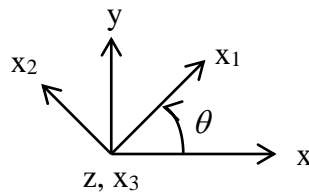


Figure 5.4: Transformation with respect to an axis

The direction cosines for these transformations are expressed as a function of the angle θ in Table 5.1:

Table 5.1: Direction cosines

		To		
		X ₁	X ₂	X ₃
From	X	$a_{11} = \text{Cos } \theta$	$a_{12} = -\text{Sin } \theta$	$a_{13} = 0$
	Y	$a_{21} = \text{Sin } \theta$	$a_{22} = \text{Cos } \theta$	$a_{23} = 0$
	Z	$a_{31} = 0$	$a_{32} = 0$	$a_{33} = 1$

5.4.1. Transformation of stresses and strains around an axis

Using the direction cosines shown in Table 5.1, we can write the relationship between the stresses in the principal plane of the material and in the overall plane of the coordinates:

$$\{\sigma\}_1 = [T_1] \{\sigma\}_x \quad (5.38)$$

Where the transformation matrix $[T_1]$ is expressed as follows:

$$[T_1] = \begin{bmatrix} m^2 & n^2 & 0 & 0 & 0 & 2mn \\ n^2 & m^2 & 0 & 0 & 0 & -2mn \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & -n & 0 \\ 0 & 0 & 0 & n & m & 0 \\ -mn & mn & 0 & 0 & 0 & (m^2 - n^2) \end{bmatrix} \quad (5.39)$$

And we used $m = \cos \theta$ and $n = \sin \theta$. Note that the transformation matrix $[T_1]$ is asymmetric.

In the same way, the transformation of the deformations can be written as follows:

$$\{\varepsilon\}_1 = [T_2] \{\varepsilon\}_x \quad (5.40)$$

Where

$$[T_2] = \begin{bmatrix} m^2 & n^2 & 0 & 0 & 0 & mn \\ n^2 & m^2 & 0 & 0 & 0 & -mn \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & -n & 0 \\ 0 & 0 & 0 & n & m & 0 \\ -2mn & 2mn & 0 & 0 & 0 & (m^2 - n^2) \end{bmatrix} \quad (5.41)$$

5.4.2. Transformation of the stiffness matrix

The expression of the transformed stiffness matrix $[\bar{C}]$ is determined from the constitutive equation (5.7) in the main reference of the material and by employing the stress and strain transformations (5.38) and (5.40)

$$\begin{cases} \{\sigma\}_1 = [C] \{\varepsilon\}_1 \\ [T_1] \{\sigma\}_x = [C] [T_2] \{\varepsilon\}_x \end{cases} \quad (5.42)$$

$$\{\sigma\}_x = [T_1]^{-1} [C] [T_2] \{\varepsilon\}_x \quad (5.43)$$

The expression (5.43) can be written:

$$\{\sigma\}_x = [\bar{C}] \{\varepsilon\}_x \quad (5.44)$$

Where we define the transformed stiffness matrix $[\bar{C}]$ as follows:

$$[\bar{C}] = [T_1]^{-1} [C] [T_2] \quad (5.45)$$

We note here that: $[T_i(\theta)]^{-1} = [T_i(-\theta)]$ ($i = 1, 2$)

For a transformation through an arbitrary angle θ around the axis (3), the transformed stiffness matrix has the form of a monoclinic material.

$$[\bar{C}] = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} & 0 & 0 & \bar{C}_{16} \\ \bar{C}_{12} & \bar{C}_{22} & \bar{C}_{33} & 0 & 0 & \bar{C}_{26} \\ \bar{C}_{13} & \bar{C}_{23} & \bar{C}_{33} & 0 & 0 & \bar{C}_{36} \\ 0 & 0 & 0 & \bar{C}_{44} & \bar{C}_{45} & 0 \\ 0 & 0 & 0 & \bar{C}_{45} & \bar{C}_{55} & 0 \\ \bar{C}_{16} & \bar{C}_{26} & \bar{C}_{36} & 0 & 0 & \bar{C}_{66} \end{bmatrix} \quad (5.46)$$

The individual terms \bar{C}_{ij} of this matrix are determined from equations (5.39), (5.41) and (5.45):

$$\bar{C}_{11} = m^4 C_{11} + 2m^2 n^2 (C_{12} + 2C_{66}) + n^4 C_{22} \quad (5.47)$$

$$\bar{C}_{12} = n^2 m^2 (C_{11} + C_{22} - 4C_{66}) + (n^4 + m^4) C_{12} \quad (5.48)$$

$$\bar{C}_{13} = m^2 C_{13} + n^2 C_{23} \quad (5.49)$$

$$\bar{C}_{16} = nm [m^2 (C_{11} - C_{12} - 2C_{66}) + n^2 (C_{12} - C_{22} + 2C_{66})] \quad (5.50)$$

$$\bar{C}_{22} = n^4 C_{11} + 2m^2 n^2 (C_{12} + 2C_{66}) + m^4 C_{22} \quad (5.51)$$

$$\bar{C}_{23} = n^2 C_{13} + m^2 C_{23} \quad (5.52)$$

$$\bar{C}_{26} = nm [n^2 (C_{11} - C_{12} - 2C_{66}) + m^2 (C_{12} - C_{22} + 2C_{66})] \quad (5.53)$$

$$\bar{C}_{33} = C_{33} \quad (5.54)$$

$$\bar{C}_{36} = mn (C_{13} - C_{23}) \quad (5.55)$$

$$\bar{C}_{44} = m^2 C_{44} + n^2 C_{55} \quad (5.56)$$

$$\bar{C}_{45} = mn (C_{55} - C_{44}) \quad (5.57)$$

$$\bar{C}_{55} = n^2 C_{44} + m^2 C_{55} \quad (5.58)$$

$$\bar{C}_{66} = n^2 m^2 (C_{11} - 2C_{12} + C_{22}) + (n^2 - m^2)^2 C_{66} \quad (5.59)$$

We note that \bar{C}_{16} , \bar{C}_{26} , \bar{C}_{36} et \bar{C}_{45} are zero for $\theta = 0$ or $\theta = 90^\circ$ because they are multiplied by the factor " nm ".

5.4.3. Transformation of the flexibility matrix

Inverting equation (5.44), we will have:

$$\{\varepsilon\}_x = [\bar{S}] \{\sigma\}_x \quad (5.60)$$

Where:

$$\begin{aligned} [\bar{S}] &= [\bar{C}]^{-1} = ([T_1]^{-1} [C] [T_2])^{-1} \\ [\bar{S}] &= [T_1]^{-1} [C]^{-1} ([T_2])^{-1} \\ [\bar{S}] &= [T_2]^{-1} [C] [T_1] \end{aligned} \quad (5.61)$$

The transformed flexibility matrix has a symmetrical shape like the transformed rigidity matrix.

$$[\bar{S}] = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{13} & 0 & 0 & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{23} & 0 & 0 & \bar{S}_{26} \\ \bar{S}_{13} & \bar{S}_{23} & \bar{S}_{33} & 0 & 0 & \bar{S}_{36} \\ 0 & 0 & 0 & \bar{S}_{44} & \bar{S}_{45} & 0 \\ 0 & 0 & 0 & \bar{S}_{45} & \bar{S}_{55} & 0 \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{36} & 0 & 0 & \bar{S}_{66} \end{bmatrix} \quad (5.62)$$

With:

$$\bar{S}_{11} = m^4 S_{11} + m^2 n^2 (2S_{12} + S_{66}) + n^4 S_{22} \quad (5.63)$$

$$\bar{S}_{12} = n^2 m^2 (S_{11} + S_{22} - S_{66}) + (n^4 + m^4) S_{12} \quad (5.64)$$

$$\bar{S}_{13} = m^2 S_{13} + n^2 S_{23} \quad (5.65)$$

$$\bar{S}_{16} = nm [m^2 (2S_{11} - 2S_{12} - S_{66}) + n^2 (2S_{12} - 2S_{22} + S_{66})] \quad (5.66)$$

$$\bar{S}_{22} = n^4 S_{11} + m^2 n^2 (2S_{12} + S_{66}) + m^4 S_{22} \quad (5.67)$$

$$\bar{S}_{23} = n^2 S_{13} + m^2 S_{23} \quad (5.68)$$

$$\bar{S}_{26} = nm \left[n^2 (2S_{11} - 2S_{12} - S_{66}) + m^2 (2S_{12} - 2S_{22} + S_{66}) \right] \quad (5.69)$$

$$\bar{S}_{33} = S_{33} \quad (5.70)$$

$$\bar{S}_{36} = 2mn(S_{13} - S_{23}) \quad (5.71)$$

$$\bar{S}_{44} = m^2 S_{44} + n^2 S_{55} \quad (5.72)$$

$$\bar{S}_{45} = mn(S_{55} - S_{44}) \quad (5.73)$$

$$\bar{S}_{55} = n^2 S_{44} + m^2 S_{55} \quad (5.74)$$

$$\bar{S}_{66} = 4n^2 m^2 (S_{11} - 2S_{12} + S_{22}) + (n^2 - m^2) S_{66} \quad (5.75)$$

The constitutive equation (5.60) can now be:

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{pmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{13} & 0 & 0 & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{23} & 0 & 0 & \bar{S}_{26} \\ \bar{S}_{13} & \bar{S}_{23} & \bar{S}_{33} & 0 & 0 & \bar{S}_{36} \\ 0 & 0 & 0 & \bar{S}_{44} & \bar{S}_{45} & 0 \\ 0 & 0 & 0 & \bar{S}_{45} & \bar{S}_{55} & 0 \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{36} & 0 & 0 & \bar{S}_{66} \end{bmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma \\ \tau \\ \tau \\ \tau \end{pmatrix} \quad (5.76)$$

5.4.4. Transversely isotropic material

The previous equations for the transformation of the stiffness and flexibility matrix concern an orthotropic material. In the case of a transversely isotropic material, these equations can be simplified using the following equalities:

$$C_{22} = C_{33}; \quad C_{12} = C_{13}; \quad C_{55} = C_{66}; \quad C_{44} = \left(\frac{C_{22} - C_{23}}{2} \right) \quad (5.77)$$

$$S_{22} = S_{33}; \quad S_{12} = S_{13}; \quad S_{55} = S_{66}; \quad S_{44} = 2(S_{22} - S_{23}) \quad (5.78)$$

5.5. Summary of the 3D Constituent Equations

The fundamental equations $\sigma - \varepsilon$ describe the response of monoclinic elastic materials as a function of the stiffness matrix \bar{C} . The equations can be written explicitly:

$$\begin{aligned}
 \sigma_x &= \bar{C}_{11} \varepsilon_x + \bar{C}_{12} \varepsilon_y + \bar{C}_{13} \varepsilon_z + \bar{C}_{16} \gamma_{xy} \\
 \sigma_y &= \bar{C}_{12} \varepsilon_x + \bar{C}_{22} \varepsilon_y + \bar{C}_{23} \varepsilon_z + \bar{C}_{26} \gamma_{xy} \\
 \sigma_z &= \bar{C}_{13} \varepsilon_x + \bar{C}_{23} \varepsilon_y + \bar{C}_{33} \varepsilon_z + \bar{C}_{36} \gamma_{xy} \\
 \tau_{yz} &= \bar{C}_{44} \gamma_{yz} + \bar{C}_{45} \gamma_{zx} \\
 \tau_{zx} &= \bar{C}_{45} \gamma_{yz} + \bar{C}_{55} \gamma_{zx} \\
 \tau_{xy} &= \bar{C}_{16} \varepsilon_x + \bar{C}_{26} \varepsilon_y + \bar{C}_{36} \varepsilon_z + \bar{C}_{66} \gamma_{xy}
 \end{aligned} \tag{5.79}$$

And depending on the flexibility matrix we can have:

$$\begin{aligned}
 \varepsilon_x &= \bar{S}_{11} \sigma_x + \bar{S}_{12} \sigma_y + \bar{S}_{13} \sigma_z + \bar{S}_{16} \tau_{xy} \\
 \varepsilon_y &= \bar{S}_{12} \sigma_x + \bar{S}_{22} \sigma_y + \bar{S}_{23} \sigma_z + \bar{S}_{26} \tau_{xy} \\
 \varepsilon_z &= \bar{S}_{13} \sigma_x + \bar{S}_{23} \sigma_y + \bar{S}_{33} \sigma_z + \bar{S}_{36} \tau_{xy} \\
 \gamma_{yz} &= \bar{S}_{44} \tau_{yz} + \bar{S}_{45} \tau_{zx} \\
 \gamma_{zx} &= \bar{S}_{45} \tau_{yz} + \bar{S}_{55} \tau_{zx} \\
 \gamma_{xy} &= \bar{S}_{16} \sigma_x + \bar{S}_{26} \sigma_y + \bar{S}_{36} \sigma_z + \bar{S}_{66} \tau_{xy}
 \end{aligned} \tag{5.80}$$

5.6. Assumptions of the Laminate Theory

The following assumptions are fundamental in the theory of laminated plates:

1. The interface between the layers is perfect (Continuity of displacements and deformations);
2. Each layer is a homogeneous material with known effective properties;
3. The properties of the individual layer can be isotropic, orthotropic, or transversely isotropic;
4. Each layer is in a plane stress state;
5. The laminate deforms according to the "Kirchhoff - Love" hypotheses for the bending and tensile strength of thin plates:

- 5.1 The normal to the middle plane remains straight and is normal to the deformed middle plane after deformation;
- 5.2 The normal on the medium plane do not change in length.

5.7. Deformation-Displacement Relationship

The first "Kirchhoff-Love" hypothesis requires that the shear deformations γ_{xz} et γ_{zy} be zero.

The second hypothesis of "Kirchhoff - Love" requires that the transverse displacement along the "Z" axis of the middle plane is a function of x and y only, i.e. $w = w(x, y)$

Figure 5.5 illustrates a diagram of the X-Z plane of a deformed average surface.

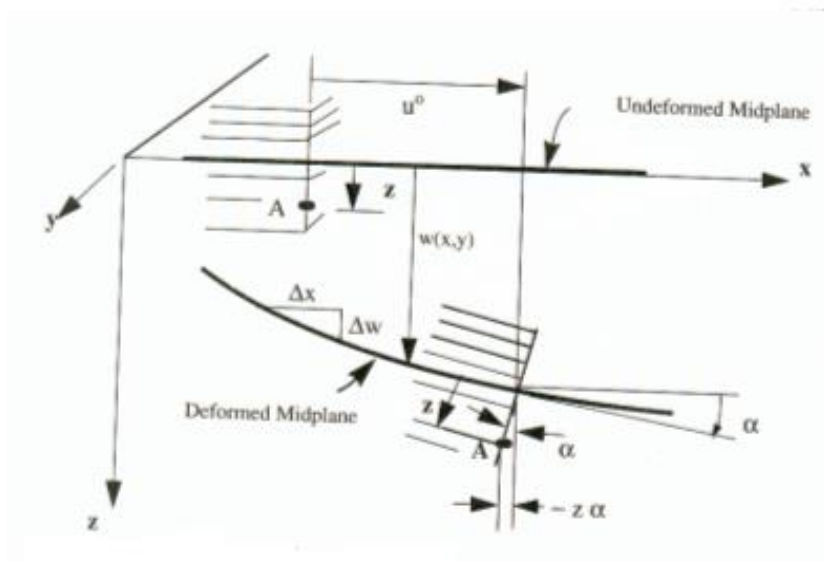


Figure 5.5: Medium plane in a deformed laminate

Suppose that the displacements are small, the rotation α of the deformed surface will also be small, which implies that:

$$\operatorname{tg} \alpha = \frac{\partial w}{\partial x} \approx \alpha \quad (5.81)$$

The total displacement " u " of point A (see Figure 5.5) can be written as the sum of the displacement of the mean plane u^0 plus the displacement due to the rotation α from the normal to the mean plane. Thus:

$$u = u^0 - z \operatorname{tg} \alpha = u^0 - z \alpha = u^0 - z \frac{\partial w}{\partial x} \quad (5.82)$$

In the same way as for the Y-Z plane, we will have:

$$v = v^0 - z \frac{\partial w}{\partial y} \quad (5.83)$$

v^0 : The displacement of the medium plane along the Y axis.

Since normals do not change lengths, the transverse displacement w is independent of "z" and can be written as follows:

$$w(x, y) = w^0(x, y) \quad (5.84)$$

Where: The upper subscript "0" represents the midrange.

The deformation-displacement equations can be written as follows:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} = \frac{\partial u^0}{\partial x} - z \frac{\partial^2 w}{\partial x^2} = \varepsilon_x^0 + z k_x \\ \varepsilon_y &= \frac{\partial v}{\partial y} = \frac{\partial v^0}{\partial y} - z \frac{\partial^2 w}{\partial y^2} = \varepsilon_y^0 + z k_y \\ \gamma_{xy} &= \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = \frac{\partial u^0}{\partial y} - 2z \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial v^0}{\partial x} = \gamma_{xy}^0 + z k_{xy} \end{aligned} \quad (5.85)$$

Where curvatures $\{k\}$ are defined as follows:

$$k_x = -\frac{\partial^2 w}{\partial x^2} \quad k_y = -\frac{\partial^2 w}{\partial y^2} \quad k_{xy} = -2 \frac{\partial^2 w}{\partial xy} \quad (5.86)$$

Combining equations (5.85) and (5.86) we will have:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \quad (5.87)$$

Or more simply:

$$\{\varepsilon_x\} = \{\varepsilon^0\}_x + z\{k\}_x \quad (5.88)$$

The last equation expresses the total deformations $\{\varepsilon_x\}$ at each dimension "z" in the laminate as a function of the deformations at the midrange plane $\{\varepsilon^0\}_x$ and the curvatures $\{k\}_x$, this is the fundamental equation of the laminate theory.

5.8. Constraints

The expression of stresses can now be determined by writing down the relationship between the latter and the strains and by using equation (5.88):

$$\{\sigma\}_x = [\bar{Q}]^k \{\varepsilon\}_x \quad (5.89)$$

Where: $[\bar{Q}]^k$ matrix of reduced stiffness transformed by the k^{th} ply corresponding to the z-dimension.

Note: A very important point in the use of the constituent equations of laminates is that the transformed matrix $[\bar{Q}]^k$ varies with the orientation of the fibers of each layer

From equations (5.88) and (5.89), we can write:

$$\{\sigma\}^k = [\bar{Q}]^k \{\varepsilon^0\} + [\bar{Q}]^k z \{k\} \quad (5.90)$$

5.9. Forces per unit length

The forces per unit length $\{N_x, N_y, N_{xy}\}$ (see Figure 5.6) are defined as follows:

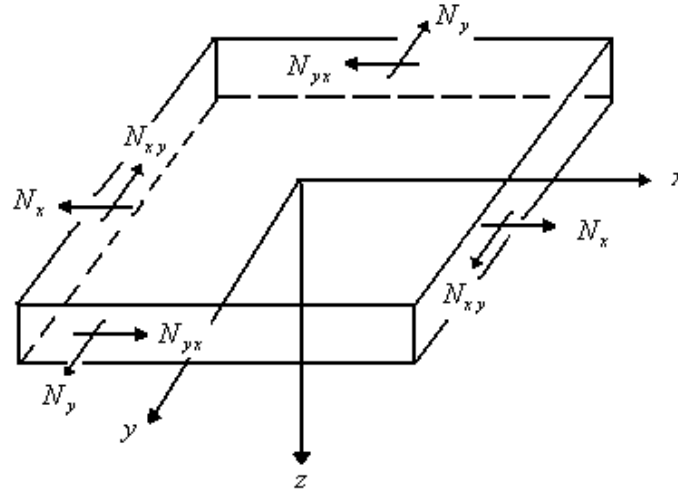


Figure 5.6: Forces per unit length

$$\begin{aligned}
 N_x &= \int_{-H}^{+H} \sigma_x dz \\
 N_y &= \int_{-H}^{+H} \sigma_y dz \\
 N_{xy} &= \int_{-H}^{+H} \tau_{xy} dz
 \end{aligned} \tag{5.91}$$

The three equations can be written in condensed form:

$$\{N\} = \int_{-H}^{+H} \{\sigma\} dz \tag{5.92}$$

From equations (5.90) and (5.91), we will have:

$$\{N\} = \int_{-H}^{+H} [\bar{Q}]^k \{\varepsilon^0\} dz + \int_{-H}^{+H} [\bar{Q}]^k \{k\} z dz \tag{5.93}$$

Or in another form

$$\{N\} = \sum_{k=1}^n \left(\int_{z_{k-1}}^{z_k} [\bar{Q}] dz \right) \{\varepsilon^0\} + \sum_{k=1}^n \left(\int_{z_{k-1}}^{z_k} [\bar{Q}]^k z dz \right) \{k\} \tag{5.94}$$

This last equation can be written in the following form:

$$\{N\} = [A] \{\varepsilon^0\} + [B] \{k\} \tag{5.95}$$

Where:

$$[A] = \sum_{k=1}^n [\bar{Q}]^k (z_k - z_{k-1}) \quad (5.96)$$

$$[B] = \frac{1}{2} \sum_{k=1}^n [\bar{Q}]^k (z_k^2 - z_{k-1}^2) \quad (5.97)$$

Note:

The matrix $[\bar{Q}]^k$ is constant through each layer but can be different from one layer to another.

Equation (5.95) relates membrane forces per unit length to the deformations of the medium plane $\{\varepsilon^0\}$, and the curvatures of the laminate $\{k\}$ to the matrices $[A]$ and $[B]$. The matrix $[A]$ represents "the membrane matrix" and the matrix $[B]$ represents "the membrane-bending coupling matrix".

5.10. Moments per unit length

Moments per unit length $\{M_x, M_y, M_z\}$ (see Figure 5.7) are defined as the integral of forces " $\sigma_i z dz$ ", as follows:

$$M_x = \int_{-H}^{+H} \sigma_x z dz \quad M_y = \int_{-H}^{+H} \sigma_y z dz \quad M_{xy} = \int_{-H}^{+H} \tau_{xy} z dz \quad (5.98)$$

Or in a condensed form:

$$\{M\} = \int_{-H}^{+H} \{\sigma\} z dz \quad (5.99)$$

Substituting equation (5.90) into equation (5.99), we will have the expression of moments per unit length:

$$\{M\} = [B]\{\varepsilon^0\} + [D]\{k\} \quad (5.100)$$

Where the bending matrix is defined as follows:

$$[D] = \frac{1}{3} \sum_{k=1}^n [\bar{Q}]^k (z_k^3 - z_{k-1}^3) \quad (5.101)$$

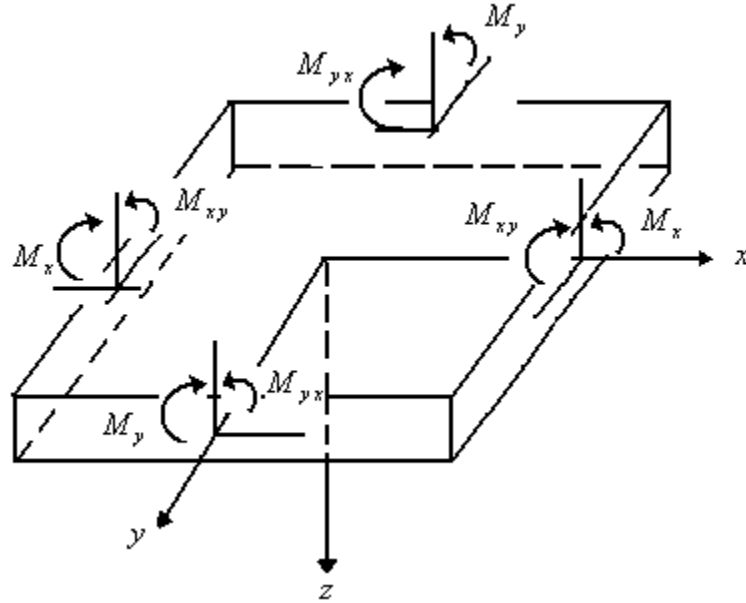


Figure 5.7: Moments per unit length

5.11. Constituent relations of the laminate

Equations (5.95) and (5.100) can be combined to give the fundamental equation of the theory of laminates:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ k \end{Bmatrix} \quad (5.102)$$

This equation can be written in a more detailed form as follows:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix} \quad (5.103)$$

Equations (5.102) or (5.103) clearly show the coupling of the flexural and membrane response of the laminate, illustrated by the matrix $[B]$. If the matrix $[B]=[0]$, then the membrane effect is decoupled from the bending effect.

Equation (5.102) can be reversed to give the following expression:

$$\begin{Bmatrix} \varepsilon^0 \\ k \end{Bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} \quad (5.104)$$

Let us return to equations (5.95) and (5.100)

$$\{N\} = [A]\{\varepsilon^0\} + [B]\{k\} \quad (5.105)$$

$$\{M\} = [B]\{\varepsilon^0\} + [D]\{k\} \quad (5.106)$$

The solution of equation (5.105) gives:

$$\{\varepsilon^0\} = [A]^{-1}\{N\} - [A]^{-1}[B]\{k\} \quad (5.107)$$

Substituting equation (5.107) into equation (5.106), we will have:

$$\{M\} = [B][A]^{-1}\{N\} + ([D] - [B][A]^{-1}[B])\{k\} \quad (5.108)$$

Equations (5.107) and (5.108) can be written in another form:

$$\begin{Bmatrix} \varepsilon^0 \\ M \end{Bmatrix} = \begin{bmatrix} A^* & B^* \\ C^* & D^* \end{bmatrix} \begin{Bmatrix} N \\ k \end{Bmatrix} \quad (5.109)$$

Where:

$$\begin{aligned} [A^*] &= [A]^{-1} \\ [B^*] &= [A]^{-1}[B] \\ [C^*] &= [B][A]^{-1} \\ [D^*] &= [D] - [B][A]^{-1}[B] \end{aligned} \quad (5.110)$$

Equation (5.109) can be written as follows:

$$\{\varepsilon^0\} = [A^*]\{N\} + [B^*]\{k\} \quad (5.111)$$

$$(5.112)$$

$$\{M\} = [C^*]\{N\} + [D^*]\{k\}$$

From equation (5.112), we can determine the expression of the curvature $\{k\}$

$$\{K\} = -[D^*]^{-1}[C^*]\{N\} + [D^*]^{-1}\{M\} \quad (5.113)$$

Substituting equation (5.113) into equation (5.111)

$$\{\varepsilon^0\} = ([A^*] - [B^*][D^*]^{-1}[C^*])\{N\} + [B^*][D^*]^{-1}\{M\} \quad (5.114)$$

Equations (5.114) and (5.113) can be written:

$$\begin{Bmatrix} \varepsilon^0 \\ k \end{Bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} \quad (5.115)$$

Where:

$$\begin{aligned} [A'] &= [A^*] - [B^*][D^*]^{-1}[C^*] \\ [B'] &= [B^*][D^*]^{-1} \\ [C'] &= -[D^*]^{-1}[C^*] \\ [D'] &= [D^*]^{-1} \end{aligned} \quad (5.116)$$

From equations (5.116) and (5.110), we will have:

$$\begin{aligned} [A'] &= [A]^{-1} + [A]^{-1}[B][D^*]^{-1}[B][A]^{-1} \\ [B'] &= -[A]^{-1}[B][D^*]^{-1} \\ [C'] &= -[D^*]^{-1}[C^*] \\ [D'] &= [D^*]^{-1} \end{aligned} \quad (5.117)$$

From the expressions of $[B']$ and $[C']$ (eq. (5.117)), we note that:

$$[C'] = [B']^T \quad (5.118)$$

Thus, equations (5.104) and (5.115) can be written as follows:

$$\begin{Bmatrix} \varepsilon^0 \\ k \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B'^T & D' \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} \quad (5.119)$$

Or in a more detailed form:

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11}' & A_{12}' & A_{16}' & B_{11}' & B_{12}' & B_{16}' \\ A_{12}' & A_{22}' & A_{26}' & B_{12}' & B_{22}' & B_{26}' \\ A_{16}' & A_{26}' & A_{66}' & B_{16}' & B_{26}' & B_{66}' \\ B_{11}' & B_{12}' & B_{16}' & D_{11}' & D_{12}' & D_{16}' \\ B_{12}' & B_{22}' & B_{26}' & D_{12}' & D_{22}' & D_{26}' \\ B_{16}' & B_{26}' & B_{66}' & D_{16}' & D_{26}' & D_{66}' \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} \quad (5.120)$$

5.12. Symmetrical laminates

For a symmetric laminate, the coupling matrix $[B]$ is zero. This can be shown by considering the contribution of two identical layers p and q (same material, same fiber orientation, and same thickness). These two layers are symmetrical with respect to the middle plane (see Figure 5.8).

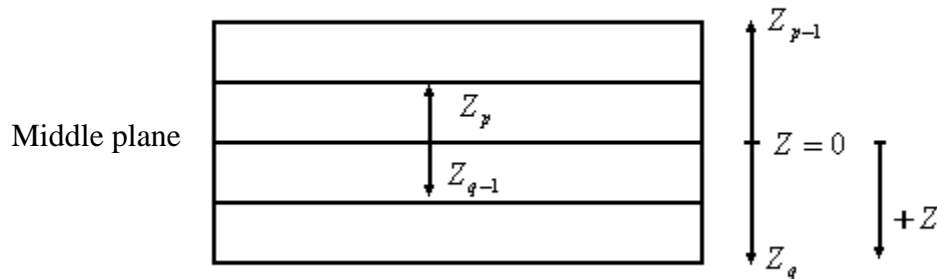


Figure 5.8: Symmetric Layers

Since the two layers are of the same material and they have the same fiber orientation (relative to the overall coordinate system) we can write that:

$$\bar{Q}_{ij(p)} = \bar{Q}_{ij(q)} \quad (5.121)$$

Given the symmetry of the two layers, we will have:

$$z_p = z_{q-1} \quad (5.122)$$

$$z_{p-1} = z_q \quad (5.123)$$

From the definition of the matrix $[B]$ (eq. (5.97)), we will have:

$$B_{ij} = \frac{1}{2} \bar{Q}_{ij(p)} (z_p^2 - z_{p-1}^2) + \frac{1}{2} \bar{Q}_{ij(q)} (z_q^2 - z_{q-1}^2) \quad (5.124)$$

By substituting equation (5.121) into equation (5.124), we obtain:

$$B_{ij(p+q)} = \frac{1}{2} \bar{Q}_{ij(p)} [z_p^2 - z_{p-1}^2 + z_{p-1}^2 - z_p^2] = 0 \quad (5.125)$$

Thus, the contribution of any two layers is zero for all terms B_{ij} .

So, the matrix $[B]$ is zero for a symmetric laminate. On the other hand, the contribution of the terms A_{ij} and D_{ij} for two symmetric layers is non-zero, i.e.:

$$A_{ij(p+q)} = 2\bar{Q}_{ij} (z_p - z_{p-1}) = 2\bar{Q}_{ij} (t_p) \neq 0 \quad (5.126)$$

$$D_{ij(p+q)} = \frac{2}{3} \bar{Q}_{ij} (z_p^3 - z_{p-1}^3) \neq 0 \quad (5.127)$$

Where: t_p is the thickness of the layer p.

The equations constituting a symmetric laminate are:

$$\{N\} = [A] \{\varepsilon^0\} \quad (5.128)$$

$$\{M\} = [D] \{k\} \quad (5.129)$$

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